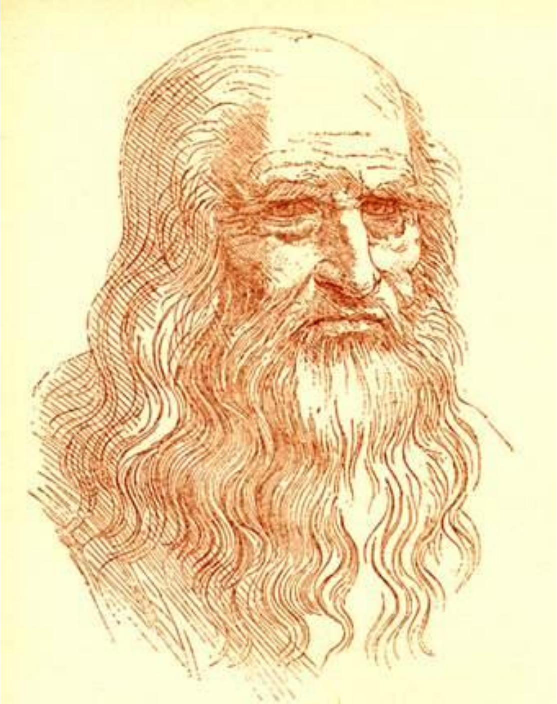
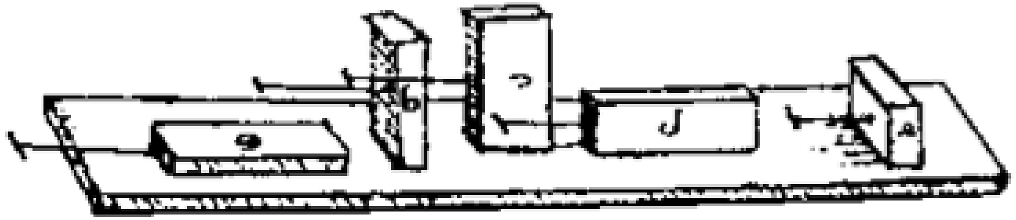
Brittle deformation and Faulting

Goal: To understand relationships between brittle failure, stresses, and fault orientation.

Frictional Strength



Question Da Vinci asked: Given that all objects shown below are of equal mass and identical shape, in which case the frictional force is greater?



Leonardo Da Vinci (1452-1519) showed that the friction force is independent of the geometrical area of contact.

Frictional Strength

- Amontons' first law: The frictional force is independent of the geometrical contact area.
- Amontons' second law: The friction force, F_S, is proportional to the normal force, F_N:

$$F_S = \mu F_N$$

$$\sigma_s = \mu \sigma_n$$

μ: static coefficient of friction

(Byerlee, 1978)

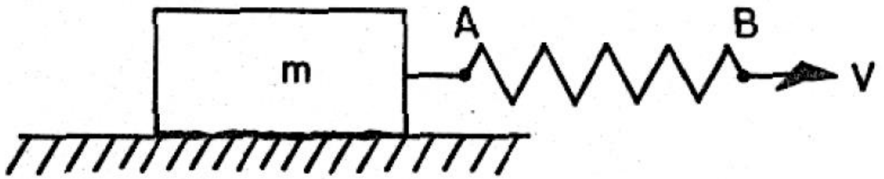
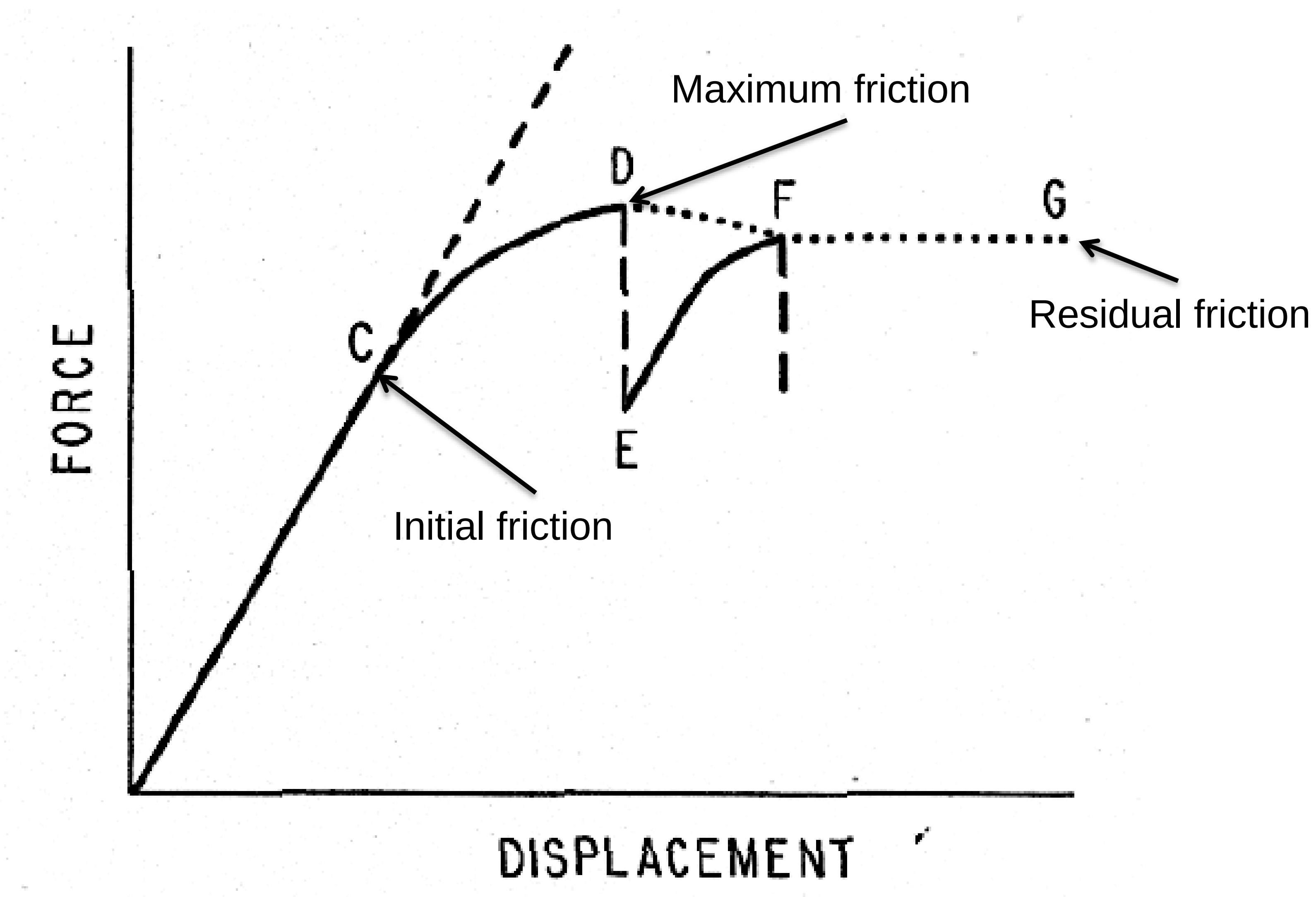


Figure 1
Schematic diagram of a typical friction experiment. For explanation see text.

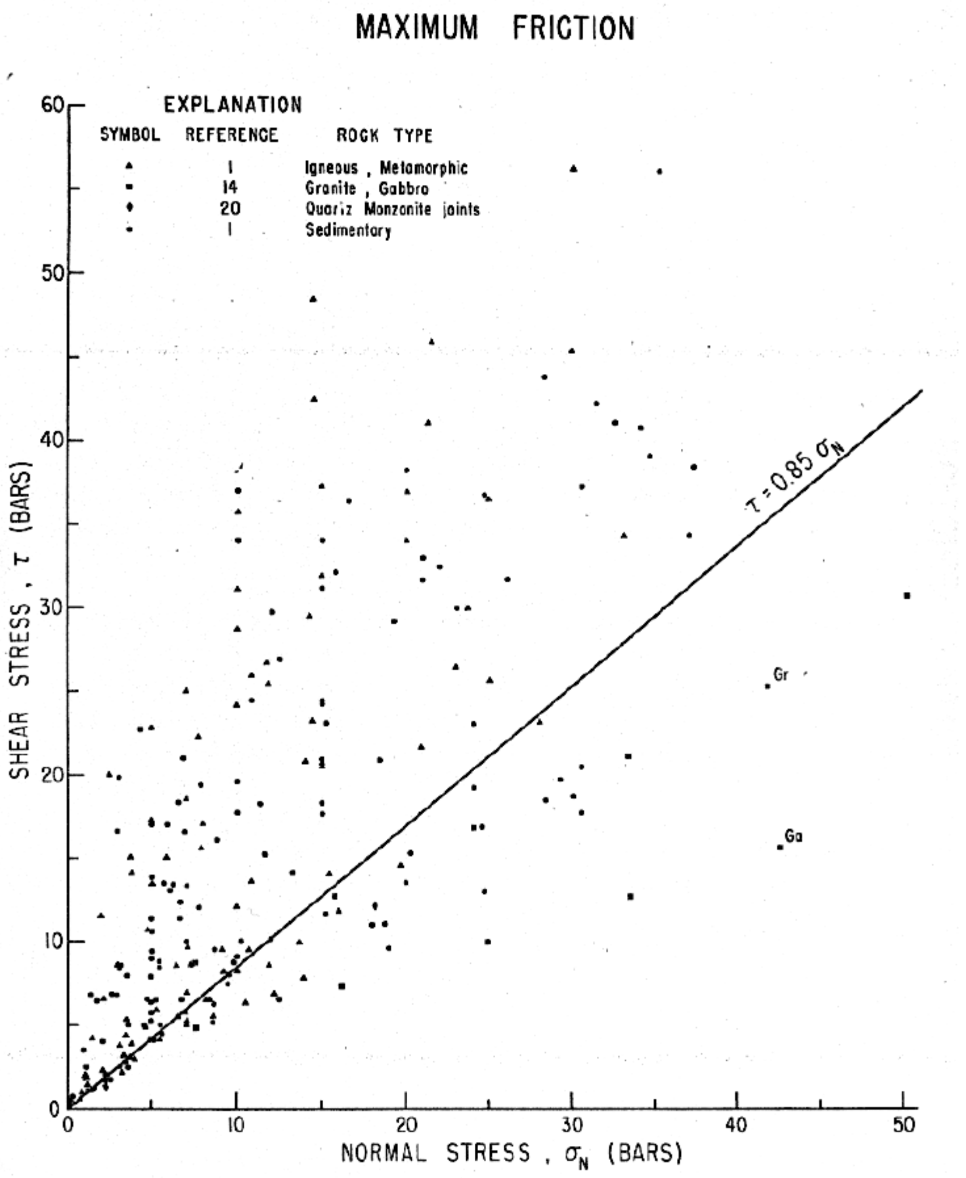
Frictional Strength



(Byerlee, 1978)

Frictional Strength

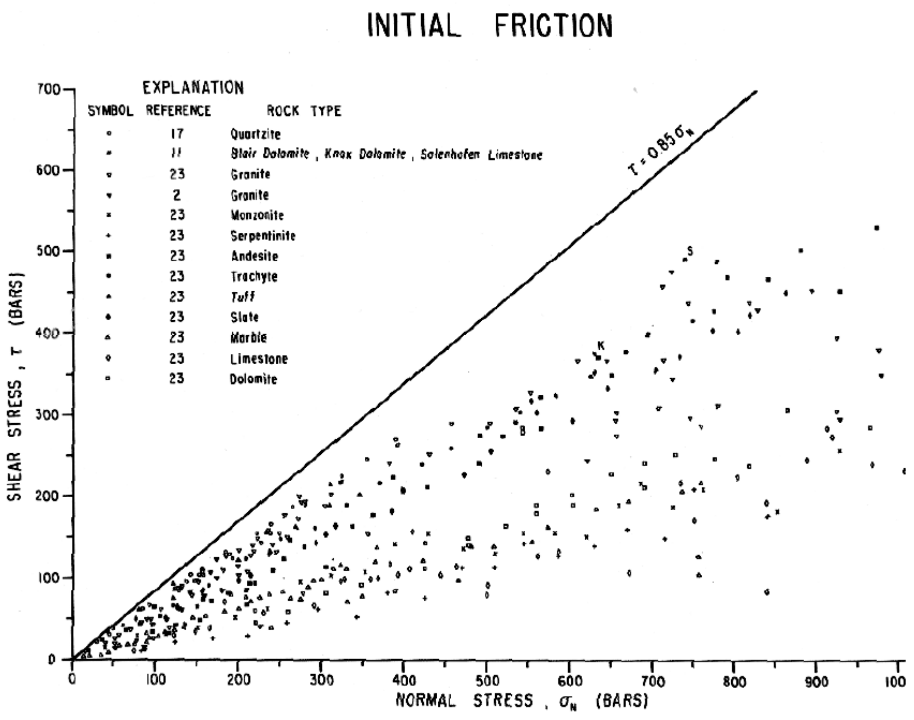
< Maximum friction for normal stress up to 50 bar >

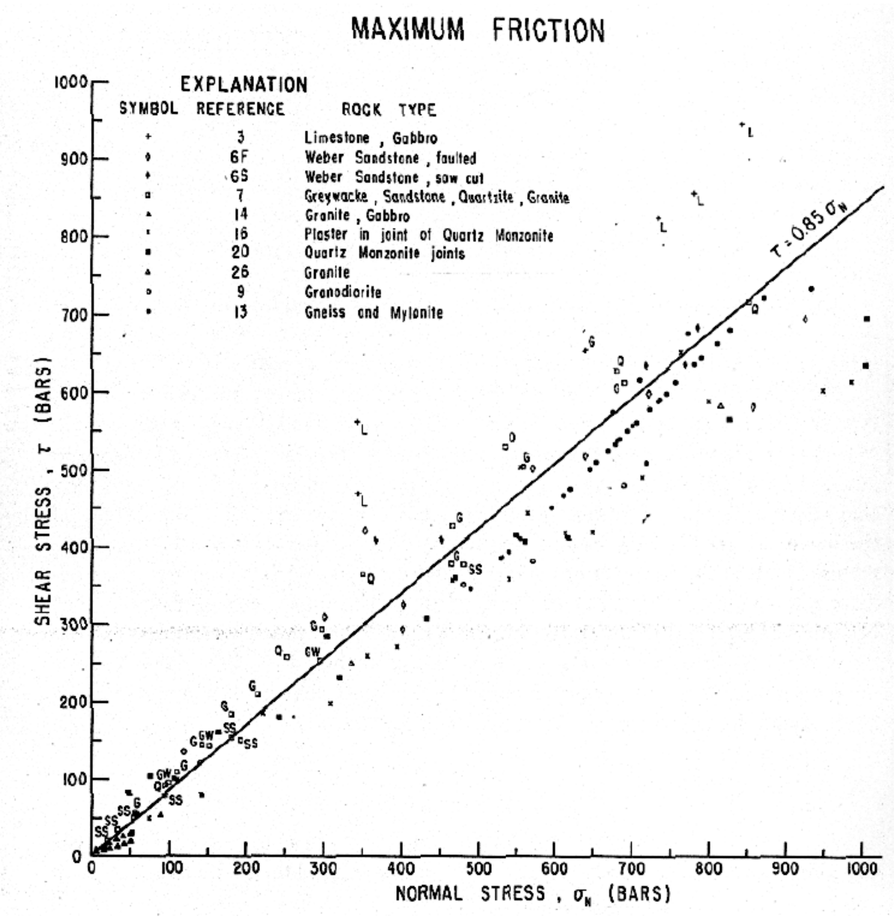


(Byerlee, 1978)

Frictional Strength

< Initial and maximum friction for normal stress up to 1 kbar >

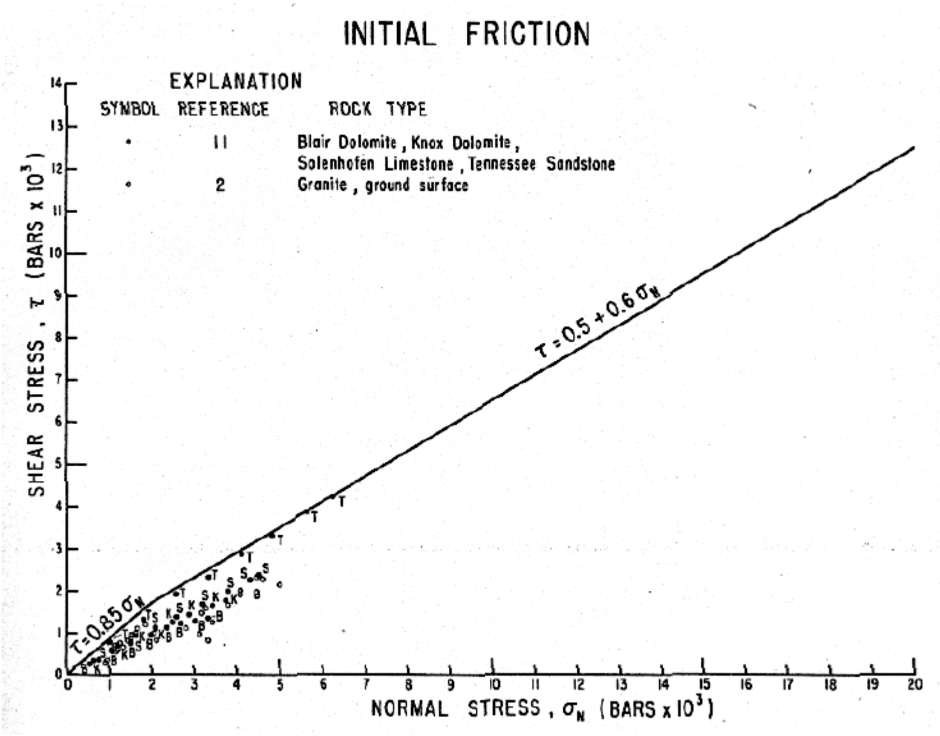


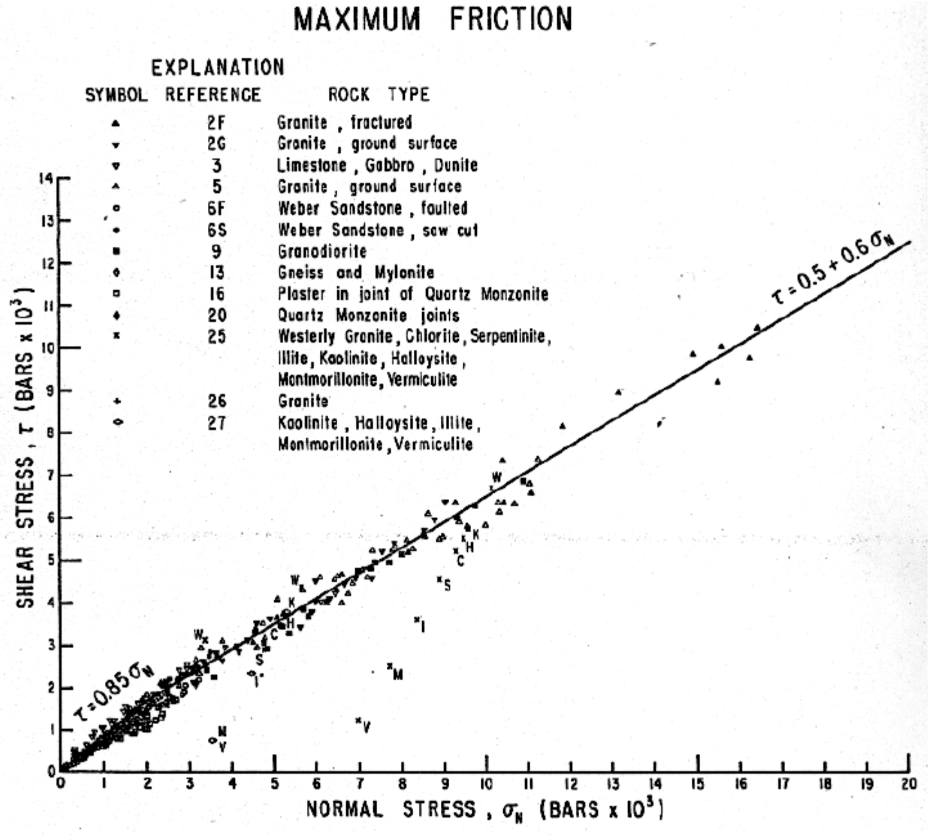


(Byerlee, 1978)

Frictional Strength

< Initial and maximum friction for normal stress up to 20 kbar >



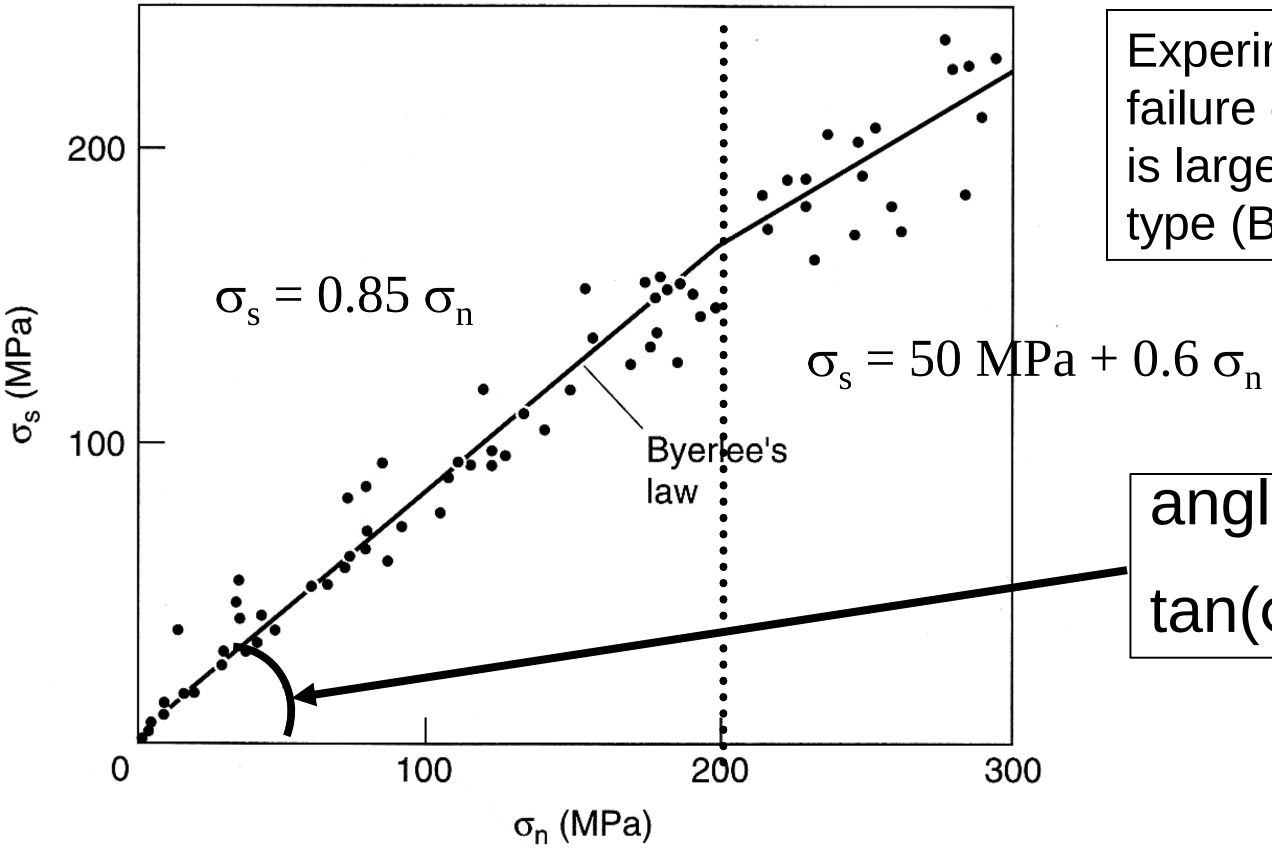


(Byerlee, 1978)

Frictional Strength

Frictional sliding criterion for most ROCKS is simple

Because of friction, certain critical shear stress is required before sliding initiates on preexisting fracture



Experimental data show that failure criterion for frictional sliding is largely independent of rock type (Byerlee, 1978)

angle of friction (φ_f):

$\tan(\varphi_f) = 0.6 \sim 0.85$

But, besides friction we have to think about how to break a surface.

Fracture Strength

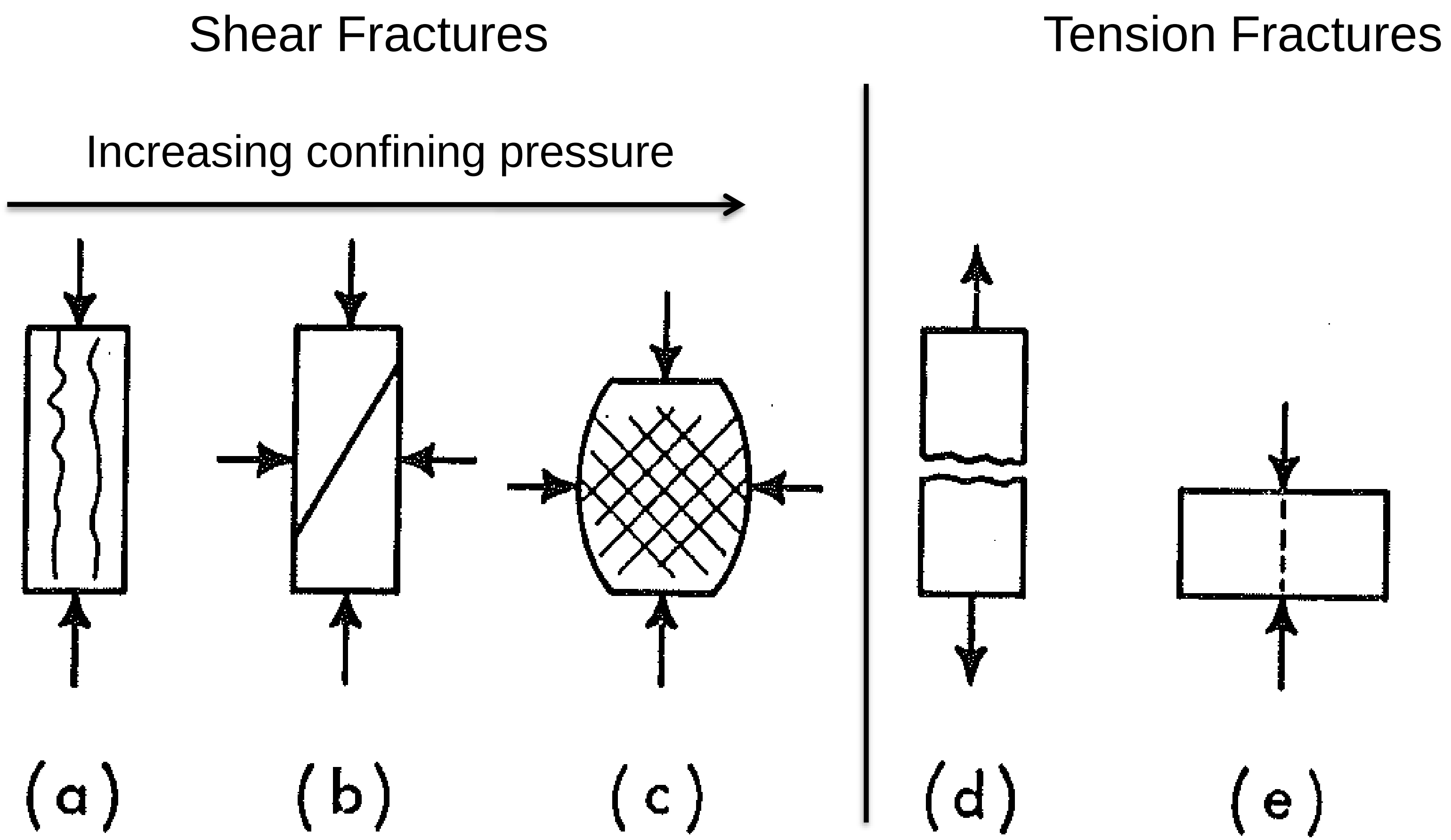
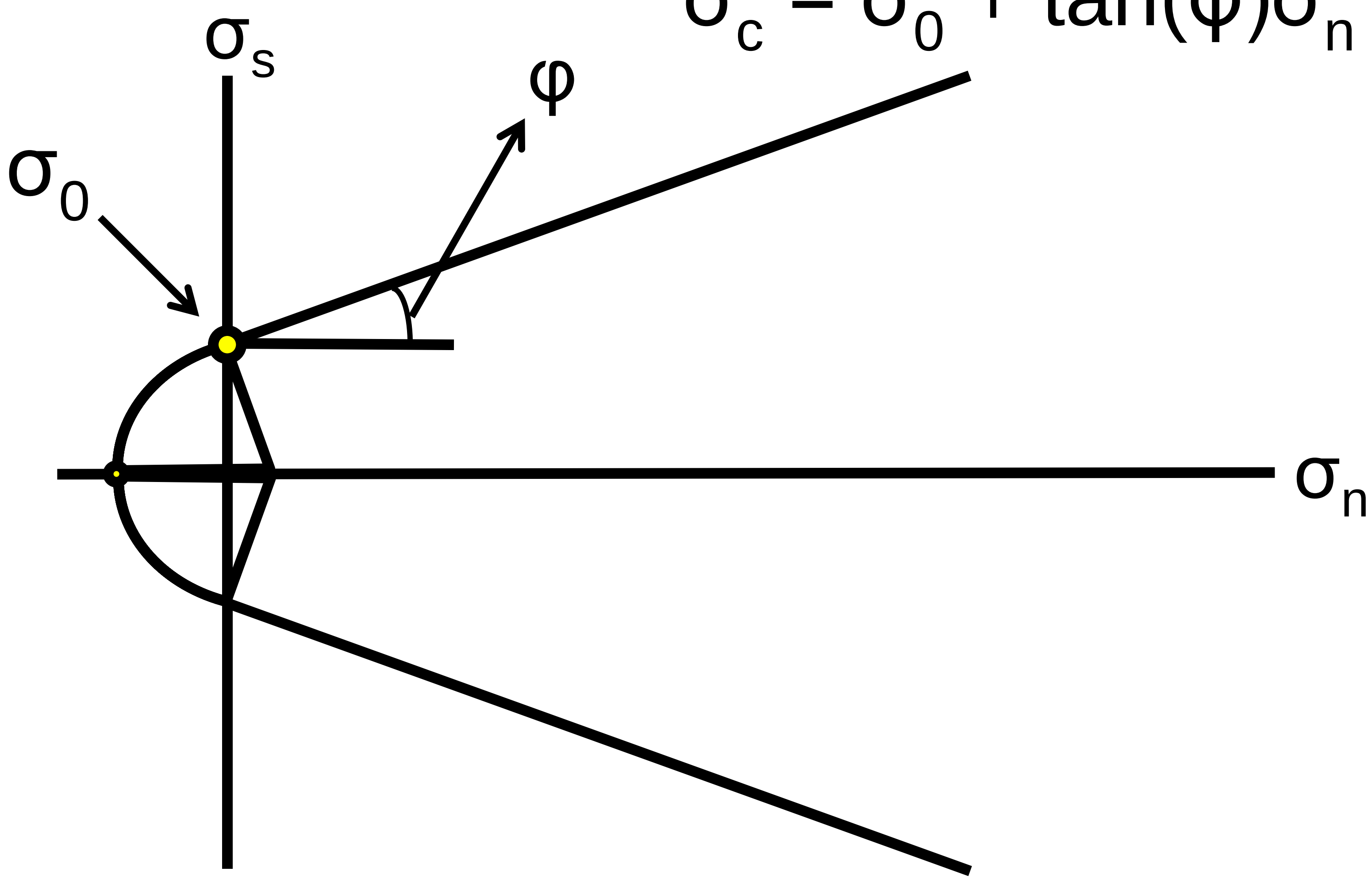


Fig. 4.5.1 (*a*) Longitudinal splitting in uniaxial compression. (*b*) Shear fracture. (*c*) Multiple shear fractures. (*d*) Extension fracture. (*e*) Extension fracture produced by line loads.

(Jaeger and Cook, 1976)

Mohr-Coulomb failure criterion

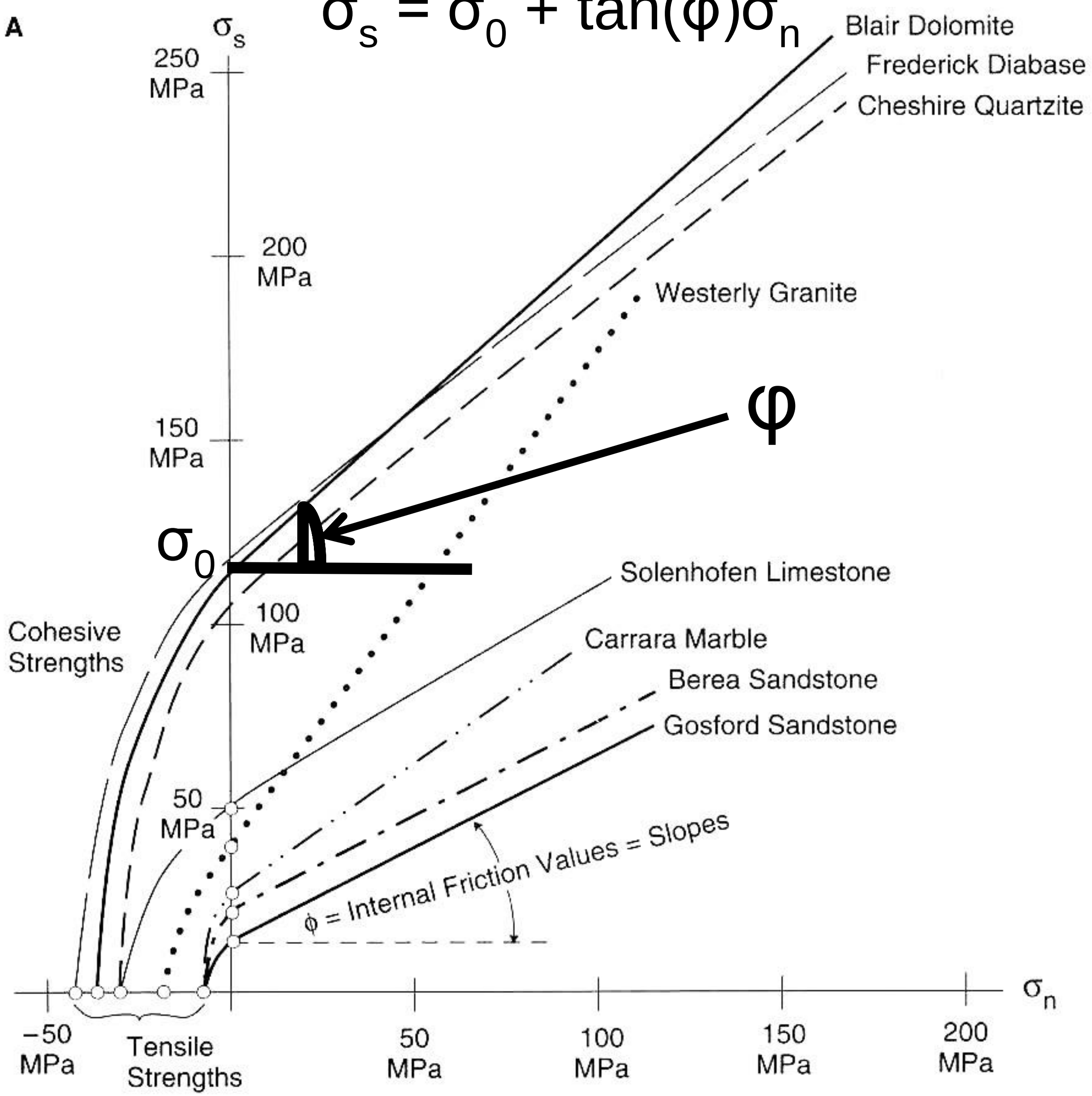
$$\sigma_c = \sigma_0 + \tan(\varphi)\sigma_n$$



Fracture Strength

< Fracture strength envelopes for different rocks >

$$\sigma_s = \sigma_0 + \tan(\varphi)\sigma_n$$



Mohr Circle: A device to relate friction and cohesion to fault *orientation*

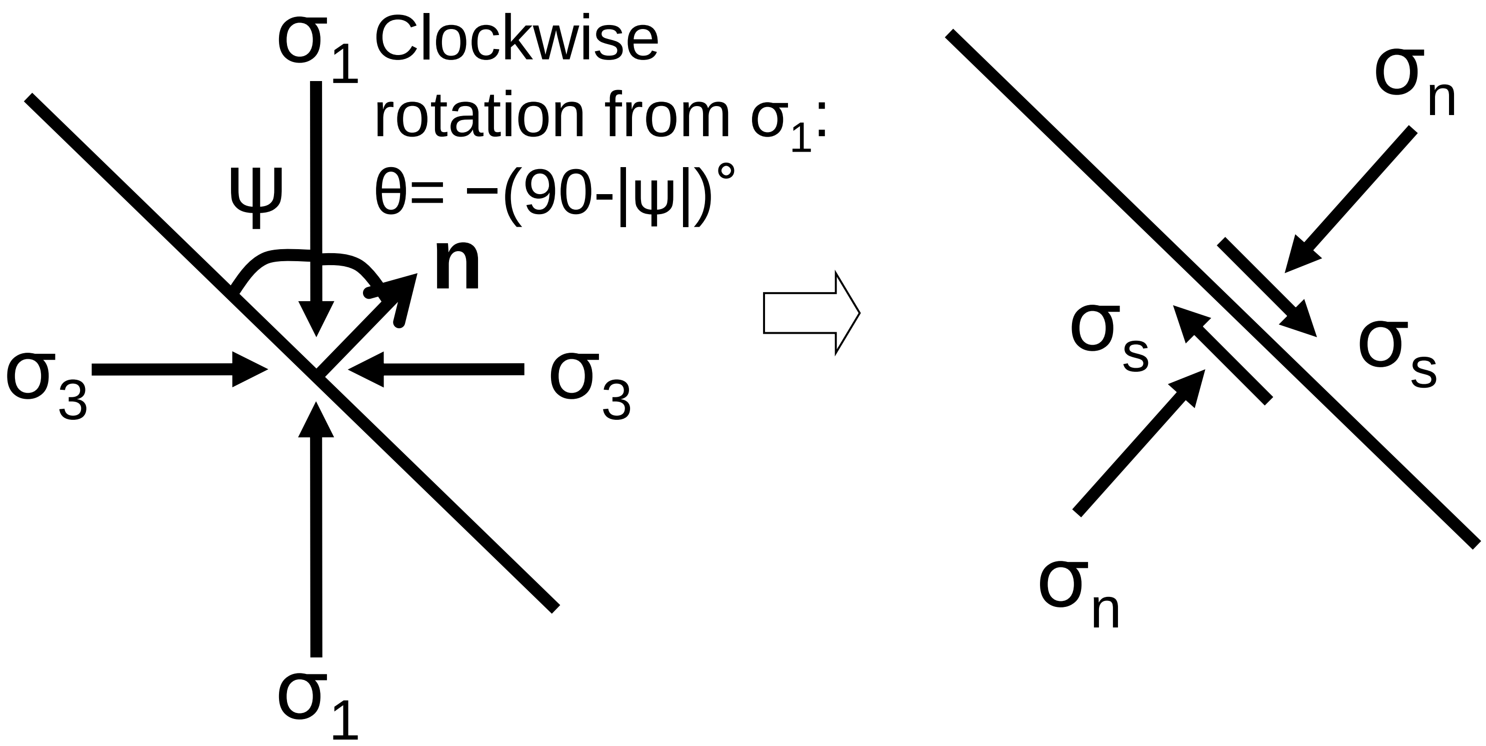
A stress review for Mohr Circle

- Stress = Force/Area
- 3 principal values, σ_1, σ_2, and σ_3, corresponding to three principal directions.
- $\sigma_1 \geq \sigma_2 \geq \sigma_3$, and positive when compressional.
- <u>(hydro/litho)static stress</u> is when $\sigma_1 = \sigma_2 = \sigma_3$
- <u>Differential stress</u> (σ_d) defined as ($\sigma_1 - \sigma_3$)

Mohr Circle

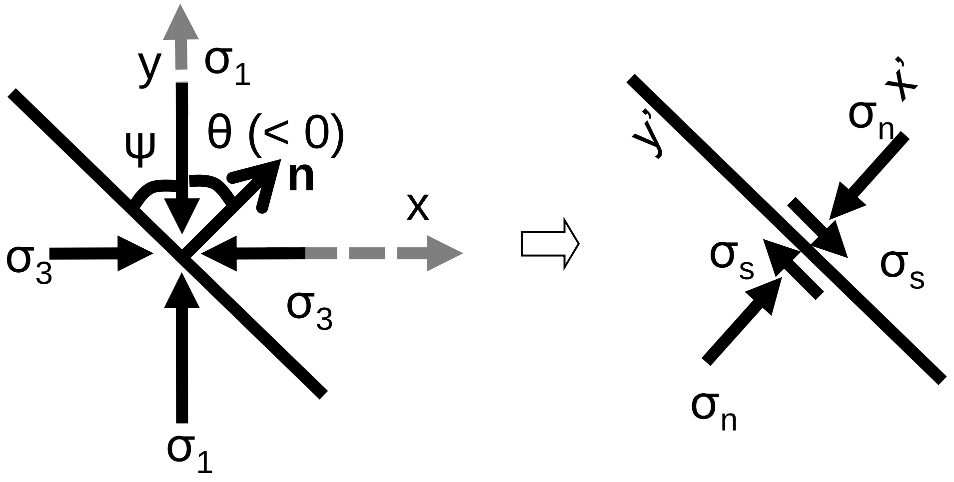
The stress is resolved into 2 components:

1. Shear stress (σ_s), acting parallel with the plane
2. Normal stress (σ_n), acting perpendicular to the plane



Mohr Circle

The stress is resolved into 2 components:



In a 2D case (e.g., plane stress), we get the *Mohr Transformation*:

$$\sigma'_{11} = \sigma_{11}\cos^2\theta + \sigma_{22}\sin^2\theta + \sigma_{12}\sin 2\theta$$
$$\sigma'_{22} = \sigma_{11}\sin^2\theta + \sigma_{22}\cos^2\theta - \sigma_{12}\sin 2\theta$$
$$\sigma'_{12} = (\sigma_{22} - \sigma_{11})\sin\theta\cos\theta + \sigma_{12}\cos 2\theta$$

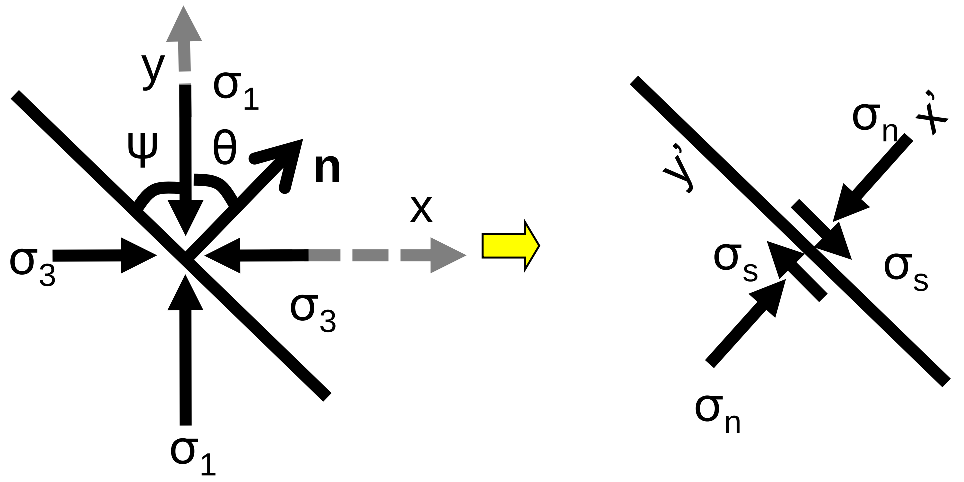
If we start with principal stresses, $\sigma_{12}=0$, $\sigma_{11}=\sigma_3$ and $\sigma_{22}=\sigma_1$. Also, $\sigma_n=\sigma'_{11}$ and $\sigma_s=\sigma'_{12}$.

Mohr Circle

- Stress components are related by:
  - $\sigma_s = ½(\sigma_1 - \sigma_3)\sin(2\theta)$
  - $\sigma_n = ½(\sigma_1 + \sigma_3) - ½(\sigma_1 - \sigma_3)\cos(2\theta)$

- From these, we get an equation for a circle:

$$[\sigma_n - ½(\sigma_1 + \sigma_3)]^2+\sigma_s=1/4(\sigma_1 - \sigma_3)^2$$

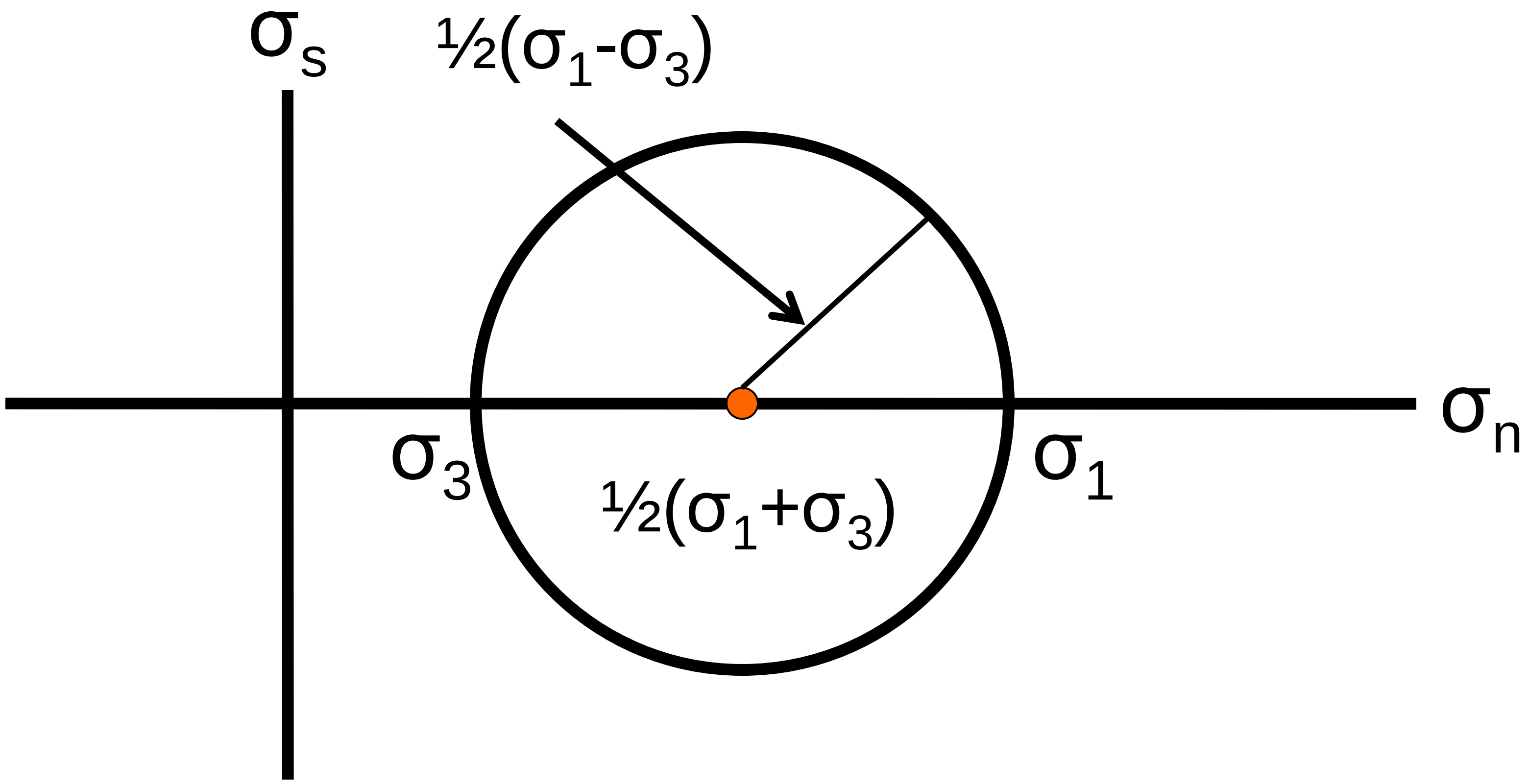


Mohr Circle

Mohr circle for stress:

Diameter = (σ_1 - σ_3), called "differential stress".

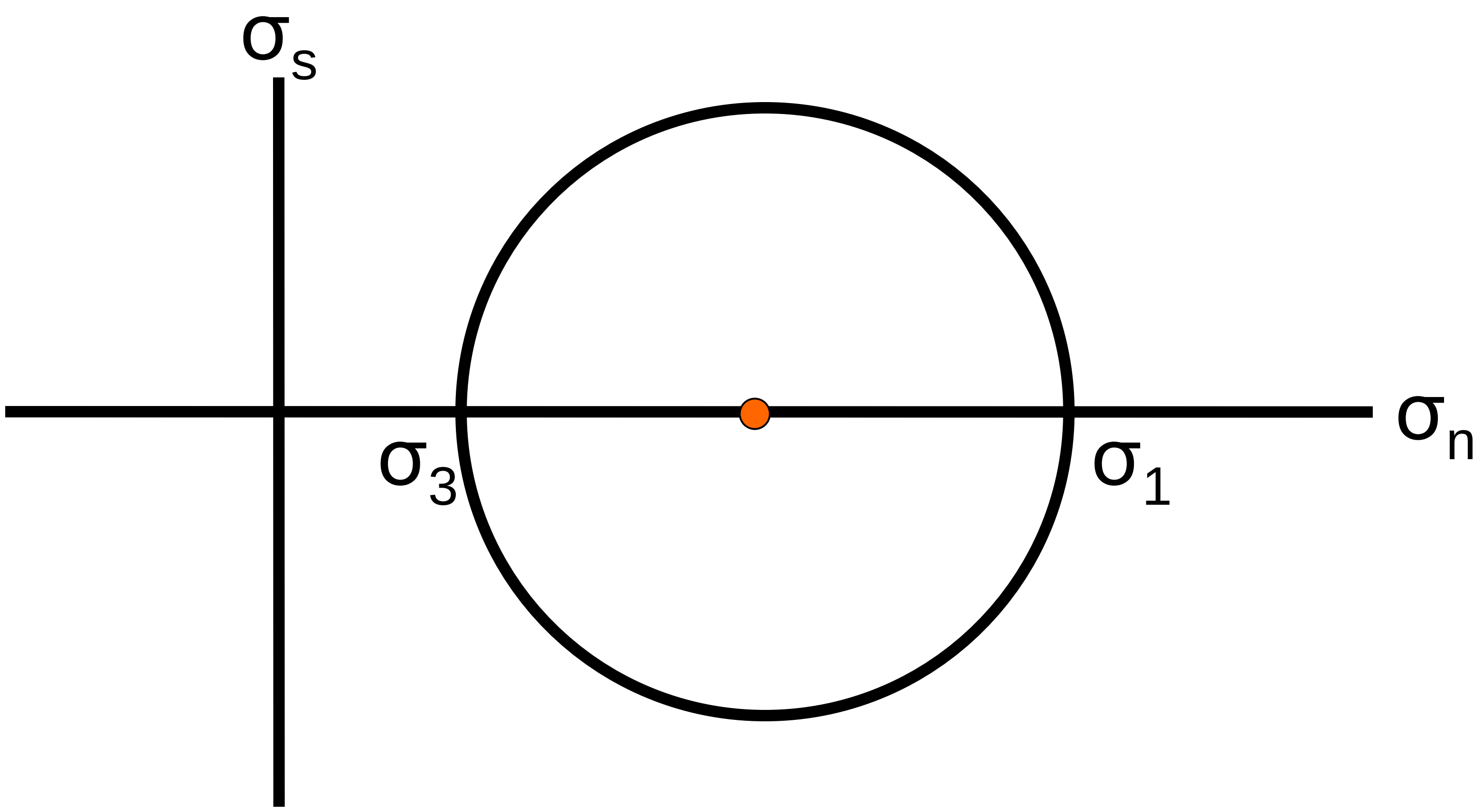
Center on the σ_n-axis at point = ½(σ_1 + σ_3)



Mohr Circle

Finding σ_s, and σ_n

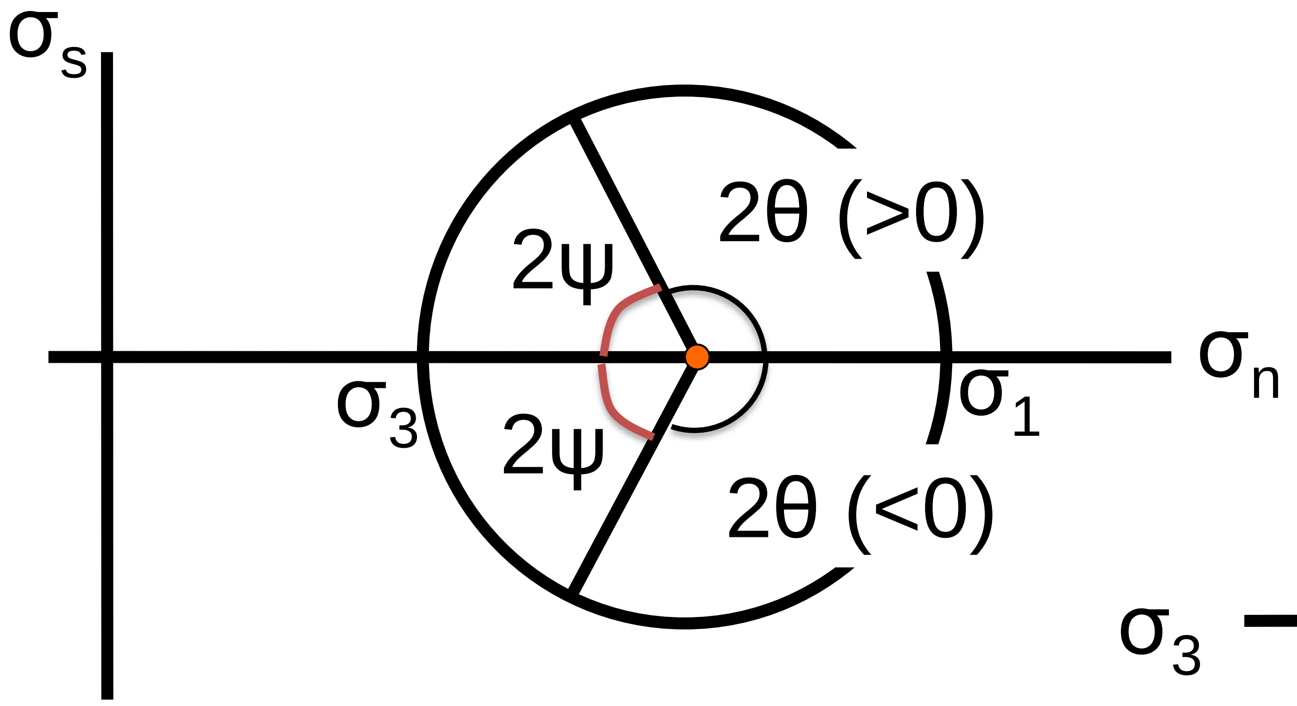
Can use a Mohr circle to find σ_s, and σ_n for any plane

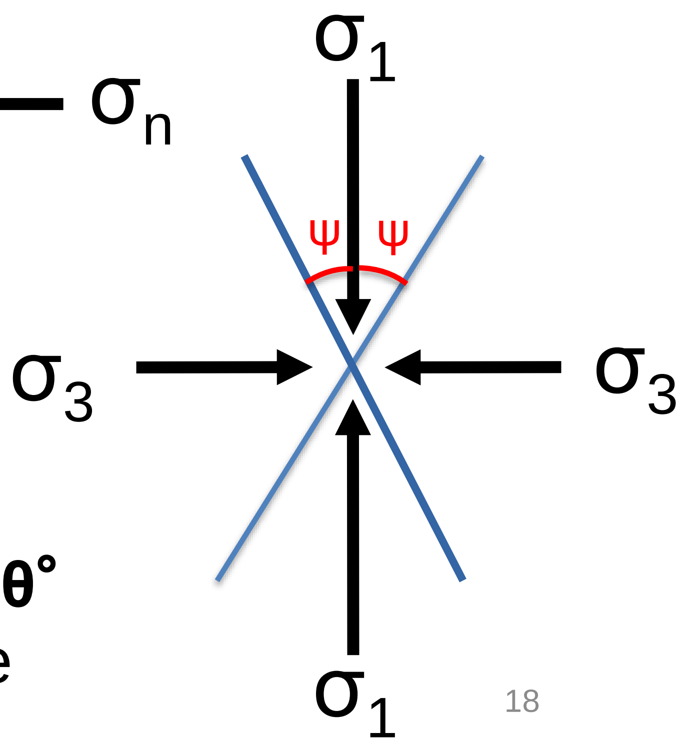


Mohr Circle

Can use a Mohr circle to find σ_s, and σ_n for any plane.

For instance, plot a line from center to edge of circle at angle **$2\theta = 180 - 2\psi°$** from σ_1.



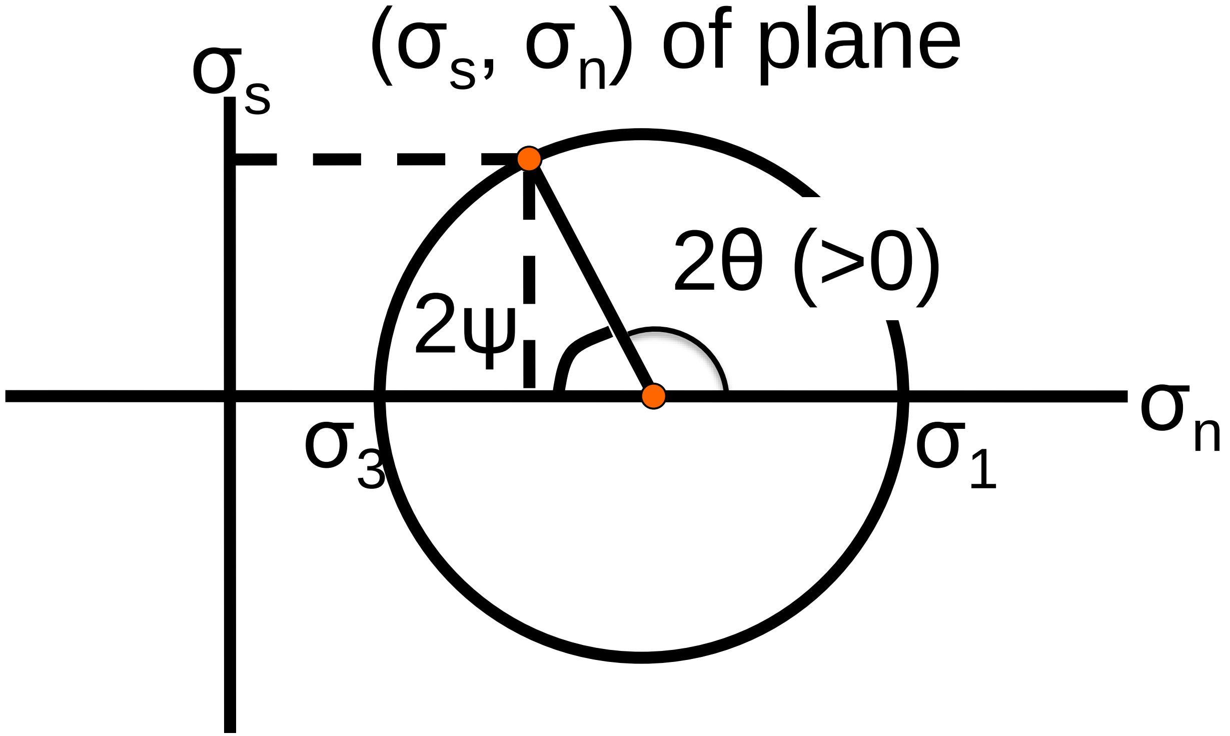


Represents **planes of which normal is $\pm\theta°$ from σ_1**. Dihedral angle of the conjugate faults is $2\psi°$.

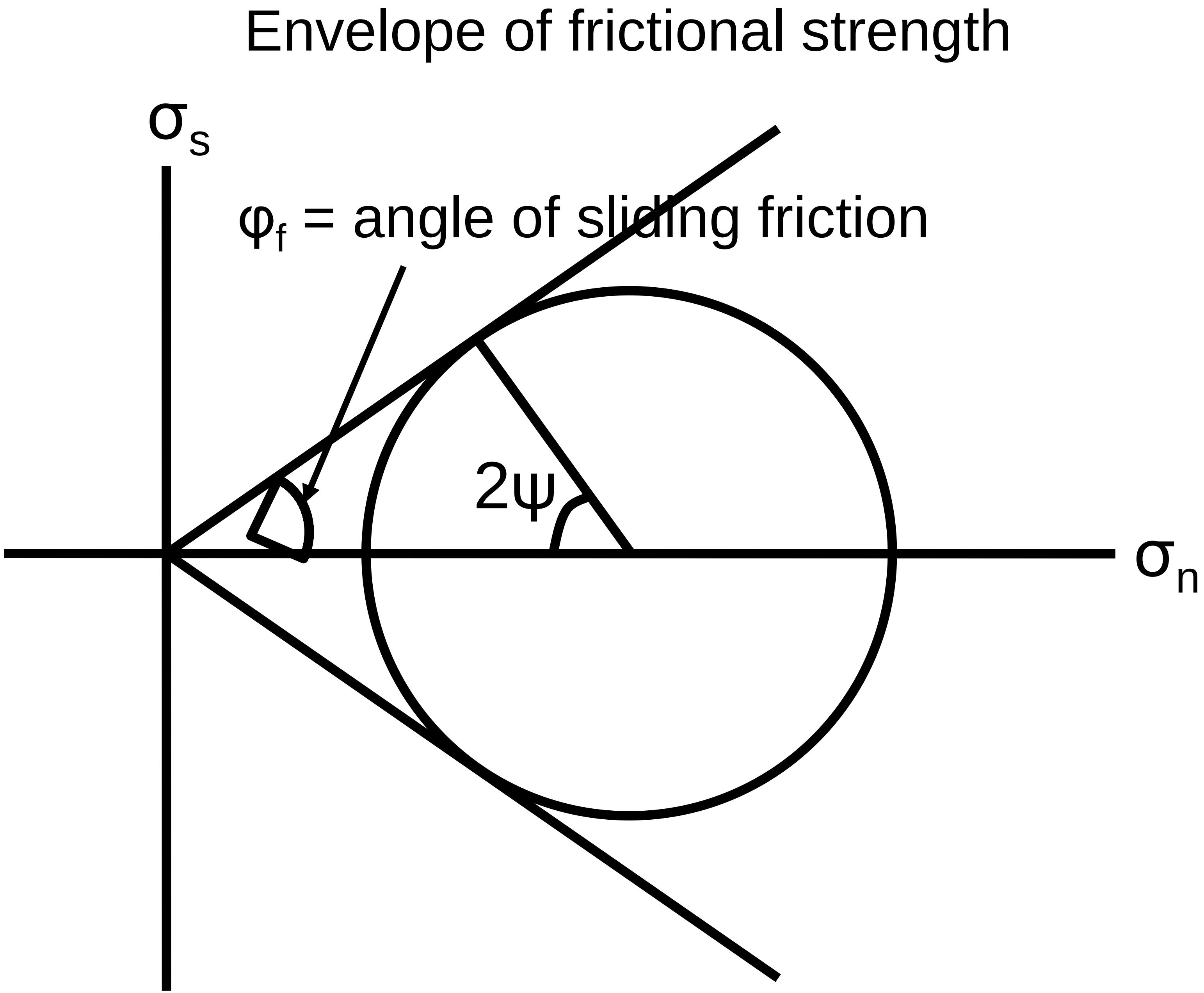
Mohr Circle

X- and y-coordinates of intersection of line and circle define σ_s and σ_n for the plane

- $\sigma_s = ½(\sigma_1 - \sigma_3)\sin(2\theta)$
- $\sigma_n = ½(\sigma_1 + \sigma_3) - ½(\sigma_1 - \sigma_3)\cos(2\theta)$,

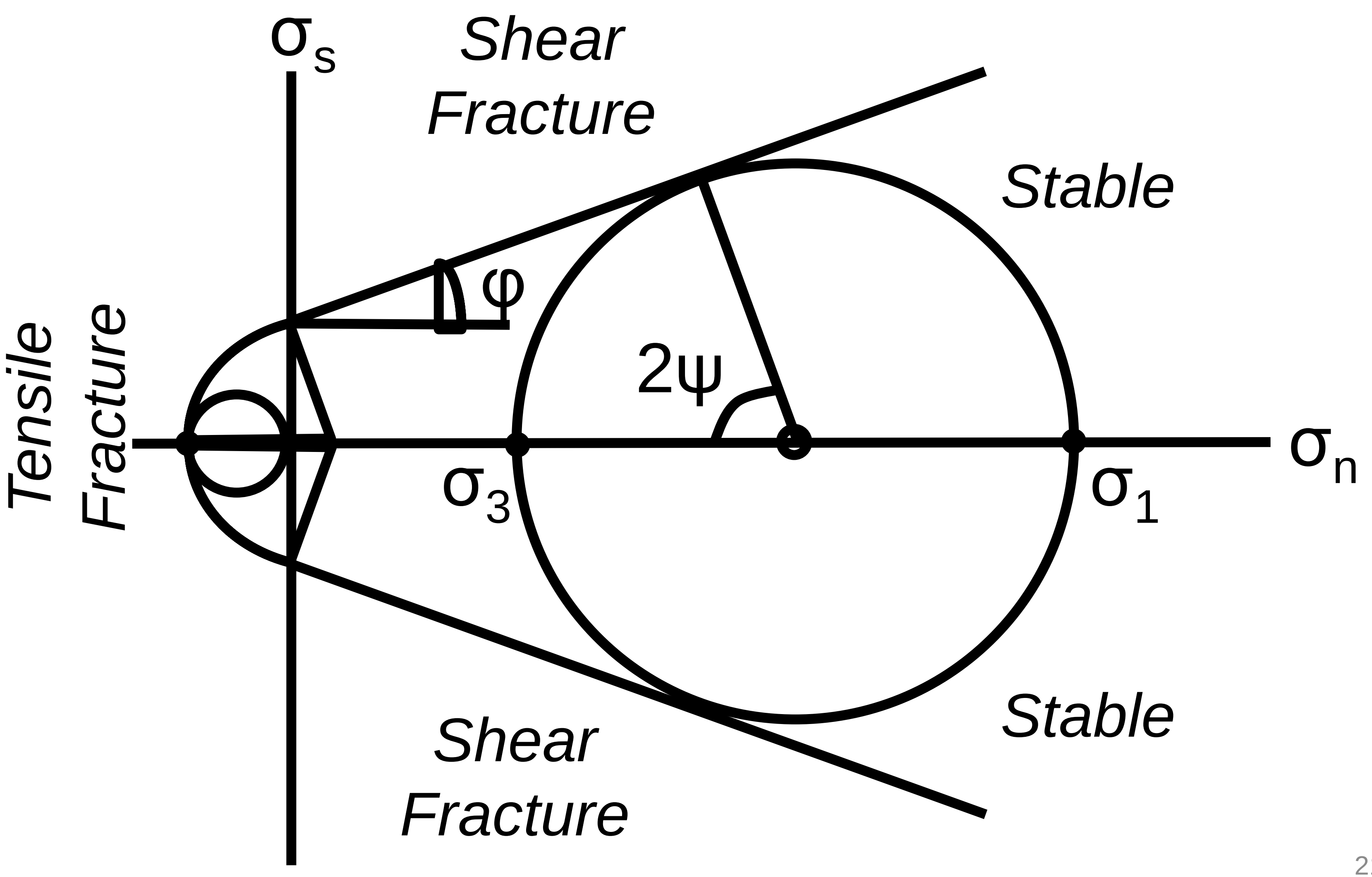


Mohr Circle and Strength Envelope



Mohr Circle and Strength Envelope

The Coulomb envelope for fracture strength



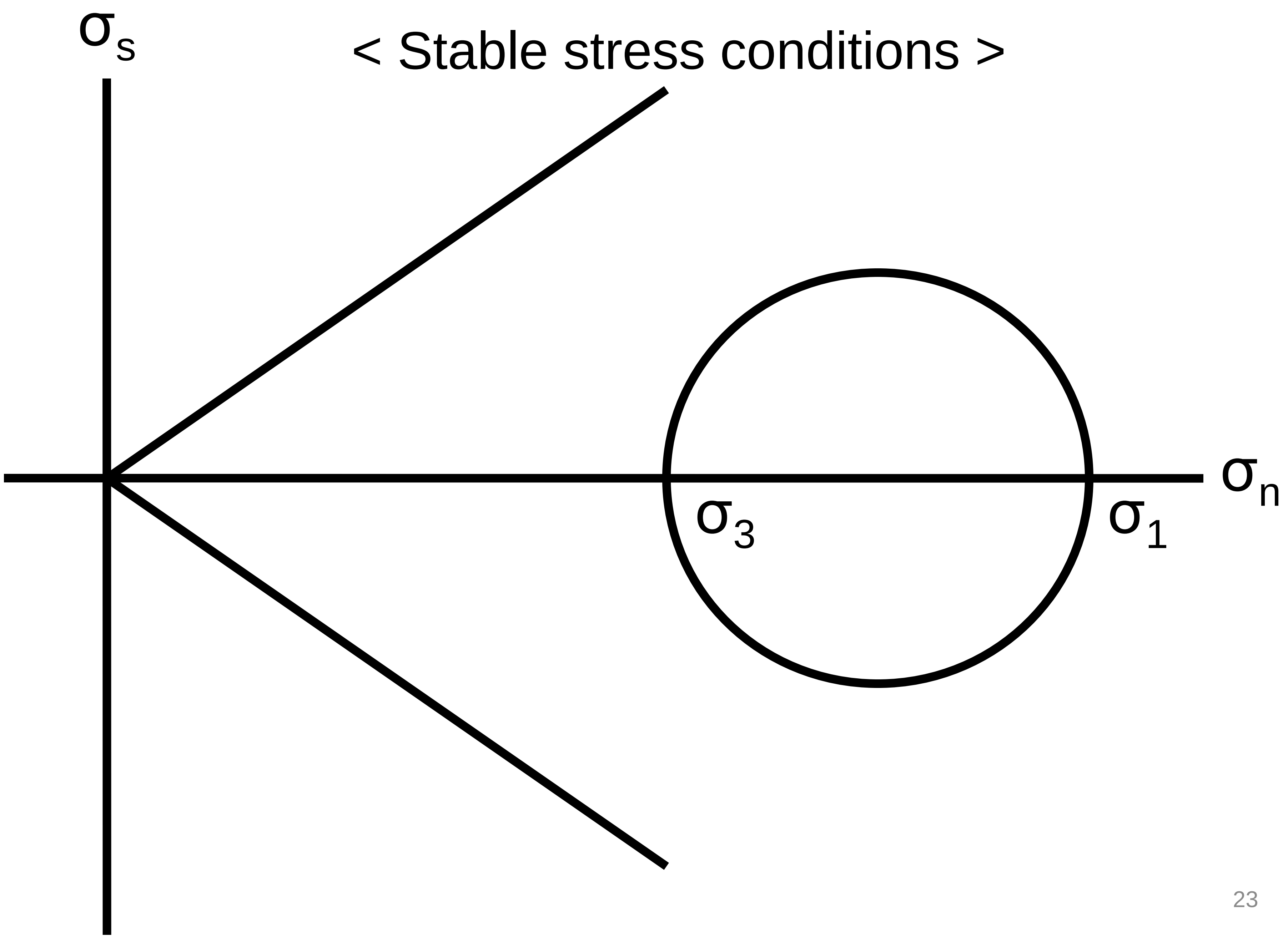
Effect of pore-fluid pressure

Pore fluid pressure (P_f) effectively lowers the stress in all directions

The effective stresses (σ_{1eff}, σ_{2eff}, and σ_{3eff}) = principal stresses - P_f

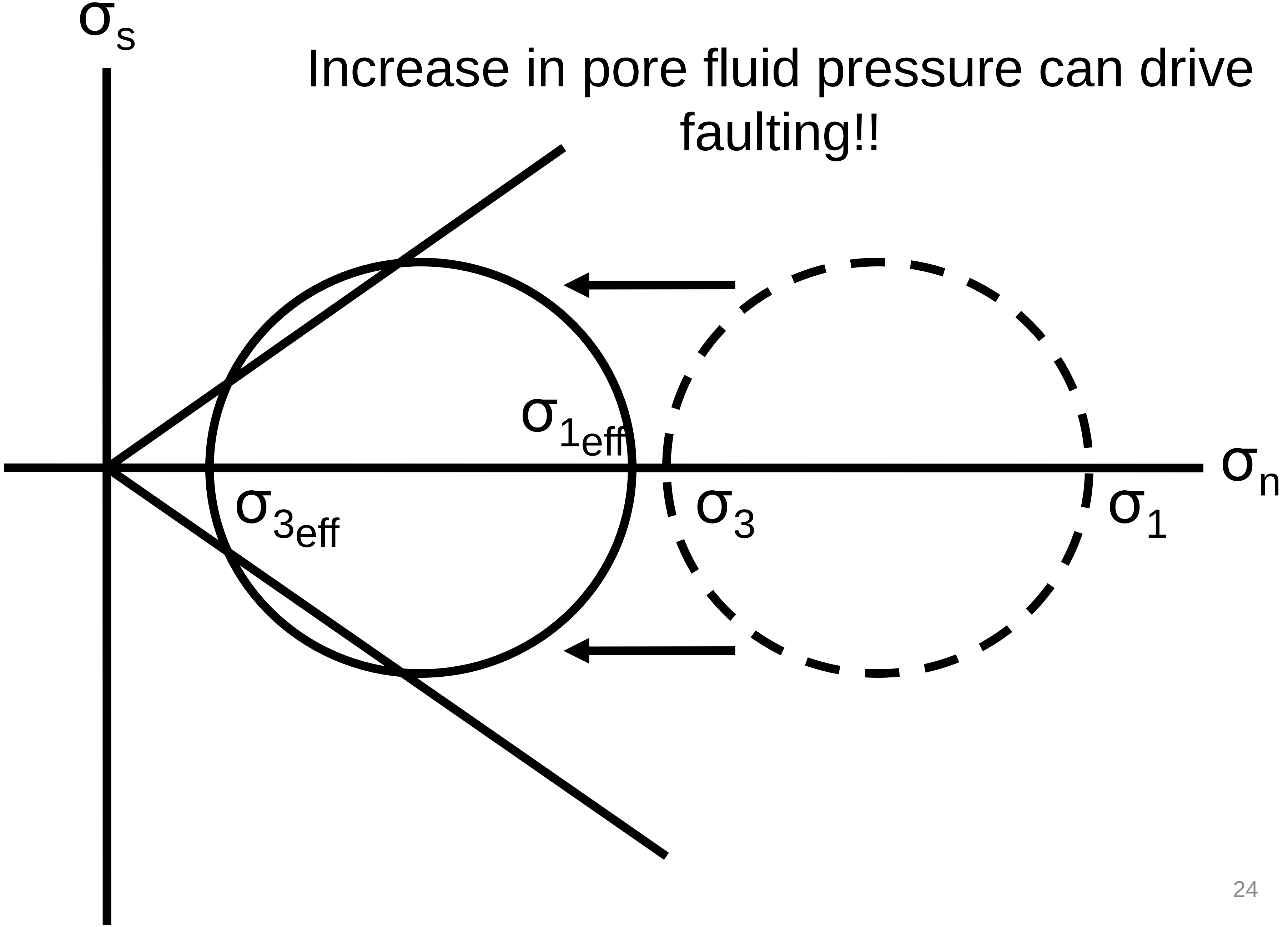
$\sigma_{1eff} = \sigma_1 - P_f$ $\sigma_{2eff} = \sigma_2 - P_f$ $\sigma_{3eff} = \sigma_3 - P_f$

Mohr Circle and Strength Envelope

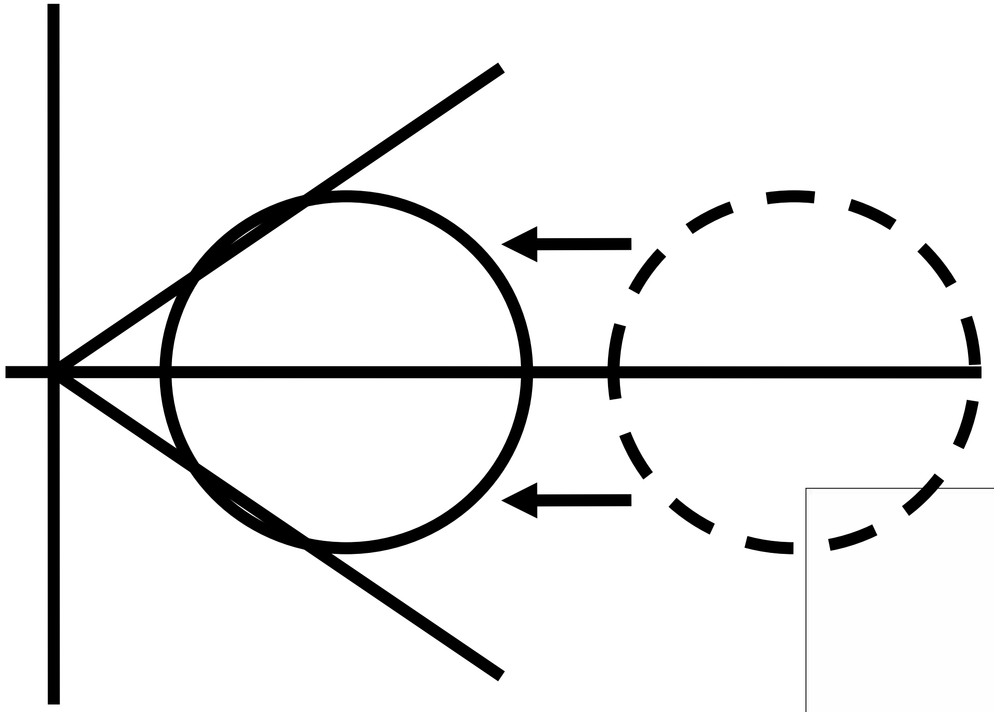


Mohr Circle and Strength Envelope

Increase in pore fluid pressure can drive faulting!!



Mohr Circle and Strength Envelope



Analogous to an air hockey table

Anderson's theory of faulting

Primary assumptions

1. Surface of the earth is not acted on by shear or normal stresses.

    - So 2 of the 3 principal stresses are parallel to the surface.

2. Homogenous rocks

3. Coulomb behavior

Anderson's theory of faulting

$$\sigma_{xx} = \rho g y + \Delta\sigma_{xx}$$

$$\sigma_{yy} = \rho g y$$

If $\Delta\sigma_{xx}$ is negative (under extension), $\sigma_1 = \sigma_{yy}$ and $\sigma_3 = \sigma_{xx}$.

Then, $\sigma_s = ½(\sigma_{yy} - \sigma_{xx})\sin(2\theta)$ and
$\sigma_n = ½(\sigma_{yy} + \sigma_{xx}) - ½(\sigma_{yy} - \sigma_{xx})\cos(2\theta)$

$$\sigma_n = \rho g y + \frac{\Delta\sigma_{xx}}{2}(1 + \cos 2\theta)$$

$$\sigma_s = -\frac{\Delta\sigma_{xx}}{2}\sin 2\theta$$

Plugging σ_n and σ_s into the Coulomb Failure criterion, $\sigma_s = \sigma_0 + f_s\sigma_n$, where $f_s = \tan(\varphi)$, we get

$$\pm\frac{\Delta\sigma_{xx}}{2}\sin 2\theta = f_s\left(\rho g y + \frac{\Delta\sigma_{xx}}{2}(1 + \cos 2\theta)\right)$$

Anderson's theory of faulting

$$\pm\frac{\Delta\sigma_{xx}}{2}\sin 2\theta = f_s\left(\rho g y + \frac{\Delta\sigma_{xx}}{2}(1+\cos 2\theta)\right)$$

In the above equation, upper sign applies to $\Delta\sigma_{xx} > 0$ (i.e., thrust faults) and the lower sign to $\Delta\sigma_{xx} < 0$ (i.e., normal faults). We can also get an expression for $\Delta\sigma_{xx}$ from this equation:

$$\Delta\sigma_{xx} = \frac{2 f_s \rho g y}{\pm\sin 2\theta - f_s(1+\cos 2\theta)}.$$

Now we are interested in the smallest possible $\Delta\sigma_{xx}$ that satisfies the above equation and the corresponding value of f_s because that value will correspond to the strength.

Anderson's theory of faulting

$$\tan 2\theta = \mp \frac{1}{f_s}$$

The upper and lower sign corresponds to thrust and normal faults, respectively. This expression simply represents the geometric relation between the failure envelope and the Mohr circle we saw earlier.

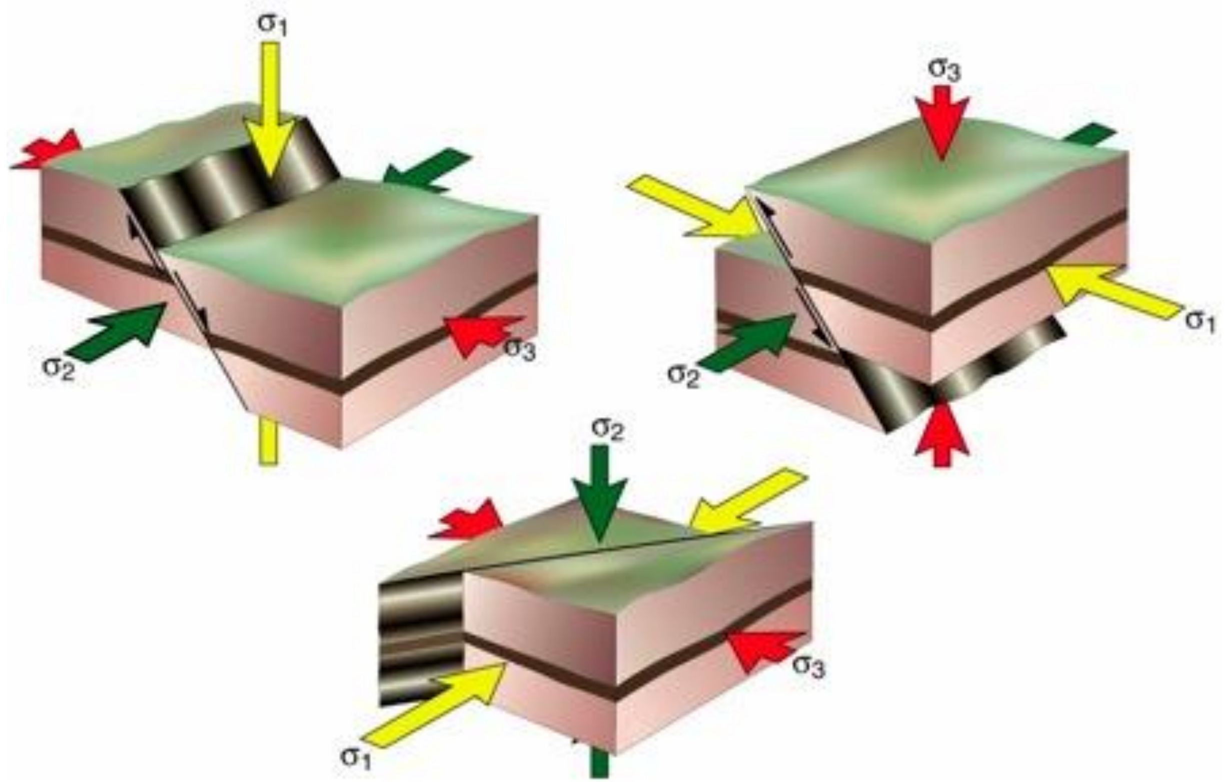
The corresponding differential stress becomes

$$\Delta\sigma_{xx} = \frac{\pm 2 f_s \rho g y}{(1 + f_s)^{1/2} \mp f_s}.$$

Anderson's theory of faulting

Most rocks have an angle of internal friction ≈ 30°

- σ_1 horizontal, σ_3 vertical — reverse faults
- σ_1 vertical, σ_3 horizontal — normal faults
- σ_1 horizontal, σ_3 horizontal — strike-slip faults



Anderson's theory of faulting

- Reverse faults (σ_1 horizontal): should form at ~30° dip
- Normal faults (σ_1 vertical): should form at ~60° dip
- Strike-slip faults: should form at ~90° dip and ~60° dihedral angle