Fold-and-thrust Belts

- Fold-and-thrust belts and submarine accretionary wedges share these features:
  - In cross-section, they occupy a **wedge-shaped deformed region** overlying a basal **detachment** or **décollement** fault.
  - The rocks or sediments beneath this fault show very little deformation.
  - The décollement fault characteristically dips toward the interior of the mountain belt or, in the case of a submarine wedge, roward the island arc
  - The topography, in contrast, slopes toward the toe or deformation front of the wedge.
  - Deformation within the wedge is generally dominated by imbricate thrust faults verging toward the toe and related fault-bend folding.

Fold-and-thrust Belts

Examples of fold-and-thrust belts and accretionary wedges: Canadian Rocky Mountains and the Lesser Antilles

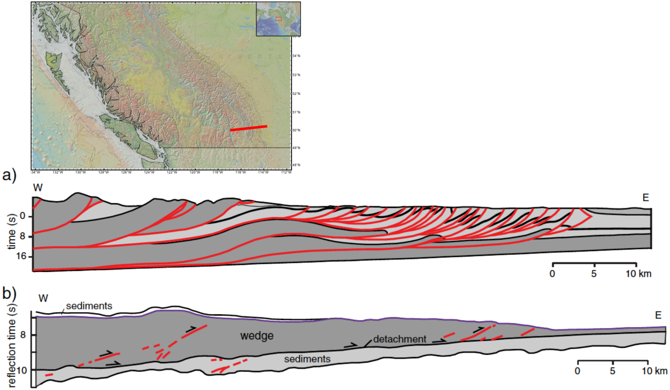
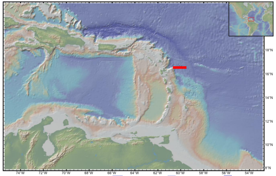


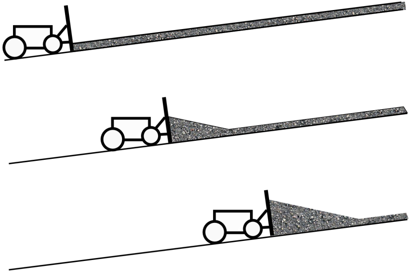
Fig. 1. a) Cross-section through the Canadian Rocky Mountains fold-and-thrust belt, redrawn from Bally et al. (1966). b) Cross-section through the Lesser Antilles acretionary wedge, redrawn from Westbrook et al. (1982).



(Buiter, Tectonophysics, 2012)

Fold-and-thrust Belts

- The starting point of the mechanical theory for these structures is the recognition that they are analogous to a wedge of sand in front of a moving bulldozer.

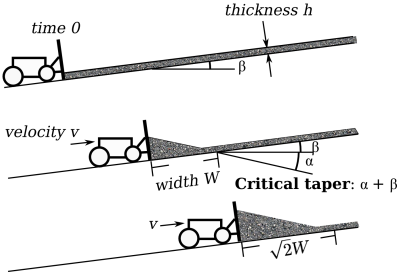


- The sand, rock, or sediment deforms until it develops a constant **critical taper**: i.e., the wedge slides stably without further deformation as it is pushed unless it encounters new fresh materials at the toe.

Fold-and-thrust Belts

- The geometry of the critical taper is governed by the relative magnitude of the basal frictional resistance to internal strength.
- An increase in the basal resistance increases the critical taper while an increase in the wedge strength decreases the critical taper.
- The state of stress within a critically tapered wedge in the upper crust is everywhere on the verge of Coulomb failure since the taper is a product of continued brittle deformation.

Mechanics of a Bulldozer Wedge: Kinematics



- $\alpha + \beta$: the critical taper.
  - α: the surface slope of a deformed wedge
  - β: the slope of a rigid hillside.
- From the mass conservation, we get the rate of the wedge growth for a constant density ρ:

$$\frac{d}{dt}\left[\frac{1}{2}\rho W^2 \tan(\alpha+\beta)\right] = \rho h v. \qquad (1)$$

Mechanics of a Bulldozer Wedge: Kinematics

- By the definition of the critical taper, $\alpha + \beta$ does not change in time. So, (1) becomes

$$W \frac{dW}{dt} = \frac{hv}{\tan(\alpha + \beta)}. \tag{2}$$

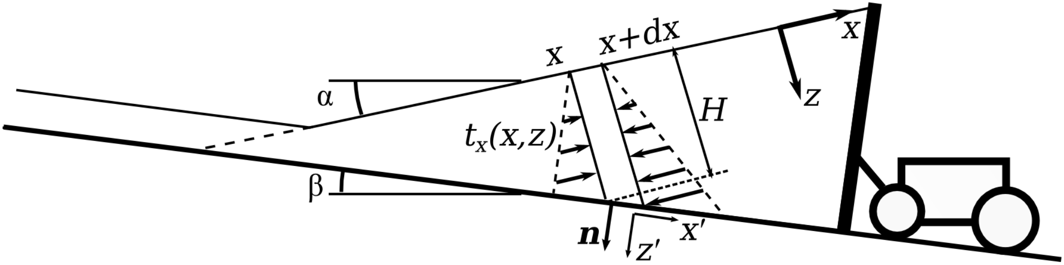
- The solution is

$$W = \left[\frac{2hv}{\tan(\alpha + \beta)} \right]^{1/2} t^{1/2} \approx \left[\frac{2hv}{\alpha + \beta} \right]^{1/2} t^{1/2}, \tag{3}$$

  where the approximation is valed if $\alpha + \beta \ll 1$ in radian.

- Since the coefficient is constant according to the assumptions we made, both the width and height($= W \tan(\alpha + \beta)$) grow like $t^{1/2}$.
- The growth is self-similar in the sense that the wedge at time $2t$ is indistinguishable from the wedge at time t, magnified $\sqrt{2}$ times.

Mechanics of a Bulldozer Wedge: Critical Taper



- In the setting described above, we consider the force balance on an infinitesimal segment of the wedge lying between x and $x + dx$.
- First, a gravitational body force whose x component per unit length along strike is

$$F_g = -(\rho H dx) g \sin\alpha, \tag{4}$$

where g is the acceleration of gravity, and H is the local wedge thickness.

Mechanics of a Bulldozer Wedge: Critical Taper

- Second, there is the net force exerted by the compressive tractions σ_{xx} acting on the sidewalls at x and $x + dx$. Setting compressive stress to be negative, we get this force as

$$\begin{aligned} F_s &= \int_0^H t_x(x) + t_x(x + dx) dz \\ &= \int_0^H (\boldsymbol{\sigma}(x) \cdot -\mathbf{e}_x)_x + (\boldsymbol{\sigma}(x + dx) \cdot \mathbf{e}_x)_x dz \\ &= \int_0^H \left[-\sigma_{xx}(x, z) + \sigma_{xx}(x + dx, z) \right] dz, \end{aligned} \tag{5}$$

where $\mathbf{e}_x = (1, 0)$ is the unit basis vector for the x axis.

Mechanics of a Bulldozer Wedge: Critical Taper

- Thirdly, and finally, there is the surface force exerted on the base.
- In a coordinate system of which x' axis is parallel to the bottom surface, we get the traction vector, $\mathbf{t}' = \boldsymbol{\sigma}' \cdot (0,1) = (\sigma_{x'z'}, \sigma_{z'z'}) \equiv (\tau_b, \sigma_n)$.
- Trasnforming $\mathbf{t}'$ to the $x - z$ coordinate system,

$$\begin{bmatrix} t_x \\ t_z \end{bmatrix} = \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix} \begin{bmatrix} \tau_b \\ \sigma_n \end{bmatrix}, \tag{6}$$

  we get

$$F_b = t_x dx = [\tau_b \cos(\alpha+\beta) - \sigma_n \sin(\alpha+\beta)]\, dx. \tag{7}$$

- The base is governed by a frictional sliding condition, $\tau_b = \mu_b(-\sigma_n)$, where μ_b is the basal friction coefficient. The sign is to make traction acting on $+x$ direction when σ_n is compressive and thus negative. With this, we have

$$F_b = -\sigma_n [\mu_b \cos(\alpha+\beta) + \sin(\alpha+\beta)]\, dx. \tag{8}$$

Mechanics of a Bulldozer Wedge: Critical Taper

- The force balance conditions is

$$F_g + F_s + F_b = 0. \tag{9}$$

- The first two forces, F_g and F_s, act in the $-x$ direction, whereas F_b acts in the $+x$ direction.
- We divide (9) with dx and assume $dx \to 0$. The result is

$$-\rho g H \sin\alpha - \sigma_n \left[\mu_b \cos(\alpha+\beta) + \sin(\alpha+\beta)\right] + \frac{d}{dx}\int_0^H \sigma_{xx} dz = 0. \tag{10}$$

- For $\alpha \ll 1$ and $\beta \ll 1$, we employ the approximations $\sin\alpha \approx \alpha$, $\sin(\alpha+\beta) \approx \alpha+\beta$, $\cos(\alpha+\beta) \approx 1$ and $\sigma_n \approx -\rho g H$. This reduces (10) to

$$\rho g H(\beta + \mu_b) + \frac{d}{dx}\int_0^H \sigma_{xx} dz \approx 0. \tag{11}$$

Mechanics of a Bulldozer Wedge: Critical Taper

- The failure criterion for non-cohesive dry sand can be written in the form

$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\phi}{1 - \sin\phi}, \tag{12}$$

  where σ_1 and σ_3 are the greatest and least principal compressive stresses, respectively, and ϕ is the internal friction angle.

  - From the last class, we know that frictional sliding satisfies $\sigma_s = \tan\phi\,\sigma_n$ (cohesion is zero since non-cohesive!) and also

$$\sigma_s = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta = \frac{\sigma_1 - \sigma_3}{2}\cos\phi, \tag{13}$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cos\phi, \tag{14}$$

    since $2\theta = \pi/2 - \phi$ under the failure condition.

  - Plugging (13) and (14) into the sliding condition, we get (12).

Mechanics of a Bulldozer Wedge: Critical Taper

- In a narrow taper (i.e., $\alpha \ll 1$ and $\beta \ll 1$), σ_1 and σ_3 are approximately horizontal and vertical:

$$\sigma_{xx} \approx \sigma_1 \approx -\frac{1+\sin\phi}{1-\sin\phi}\rho g z, \tag{15}$$

$$\sigma_{zz} \approx \sigma_3 \approx -\rho g z. \tag{16}$$

- The traction acting on the sidewalls then reduces in this approximation to

$$\frac{d}{dx}\int_0^H \sigma_{xx} dz \approx -\frac{1+\sin\phi}{1-\sin\phi}\rho g H(\alpha+\beta), \tag{17}$$

where we have used the relation $dH/dx = \tan(\alpha+\beta) \approx \alpha+\beta$.

Mechanics of a Bulldozer Wedge: Critical Taper

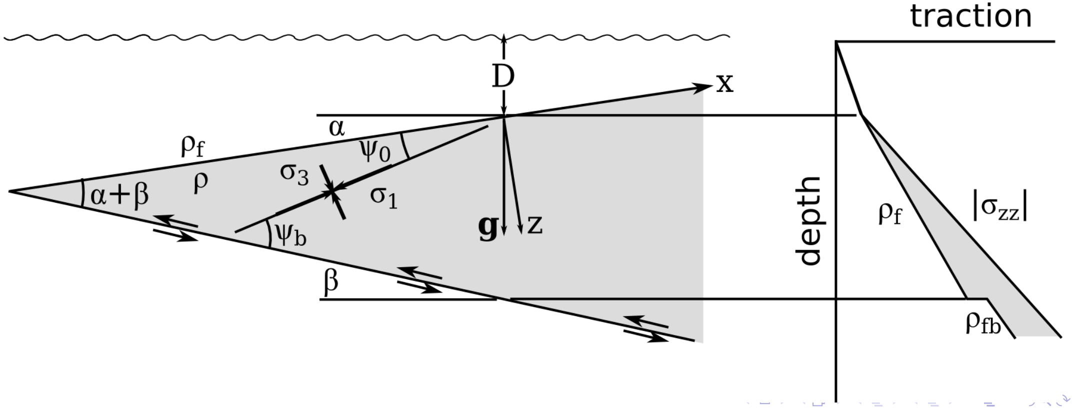
- Substituting (17) into (11), we obtain the approximate critical taper equation for a dry sand wedge in front of a bulldozer:

$$\alpha + \beta \approx \frac{1 + \sin\phi}{1 - \sin\phi}(\beta + \mu_b). \tag{18}$$

- The critical taper, $\alpha + \beta$ is proportional to the basal friction coefficient, μ_b; inversely to the internal friction angle, ϕ.

Noncohesive Coulomb Wedge

- Now, we want to consider the effects of pore fluid pressure on the stability of the wedge.
- Also, we want to get exact solutions.
- Here is the problem setting:



Noncohesive Coulomb Wedge

- The static equilibrium equation is

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} - \rho g z \sin\alpha = 0, \tag{19}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g z \cos\alpha = 0. \tag{20}$$

- Boundary conditions on the upper surface of the wedge:

$$\sigma_{xz} = 0, \quad \sigma_{zz} = -\rho_f g D, \tag{21}$$

  where D is the water depth.

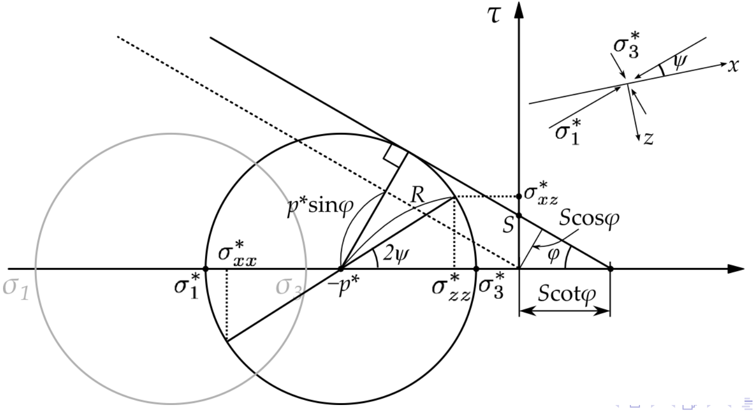
- It is convenient to define the generalized Hubbert-Rubey pore fluid to lithostatic pressure ratio by

$$\lambda = \frac{p_f - \rho_f g D}{-\sigma_{zz} - \rho_f g D}. \tag{22}$$

- We assume that λ, ρ and the internal friction coefficient (μ) are all constant.

Noncohesive Coulomb Wedge

- The density ρ is actually defined as $\rho = (1 - \eta)\rho_s + \eta\rho_f$. Since the rock ($\rho_s$) and fluid density ($\rho_f$) are constant, the constant aggregate density (i.e. constant ρ) implies a constant porosity, η.
- For convenience, we need to work out some more relations between stress components and other parameters.



Noncohesive Coulomb Wedge

- From geometry,

$$p^* = -\frac{1}{2}(\sigma^*_{zz} + \sigma^*_{xx}), \tag{23}$$
$$R = p^* \sin\phi + S\cos\phi, \tag{24}$$
$$\sigma^*_{xx} = -p^* - R\cos 2\psi, \tag{25}$$
$$\sigma^*_{zz} = -p^* + R\cos 2\psi, \tag{26}$$
$$\sigma^*_{xz} = R\sin 2\psi. \tag{27}$$

- From these relations, we derive the following expressions for later uses:

$$\frac{\sigma^*_{zz} - \sigma^*_{xx}}{2} = -\frac{\sigma^*_{zz} + \sigma^*_{xx}}{2}\sin\phi\cos 2\psi + S\cos\phi\cos 2\psi$$
$$\Rightarrow \frac{\sigma^*_{zz} - \sigma^*_{xx}}{2} = \frac{S\cot\phi - \sigma^*_{zz}}{\csc\phi\sec 2\psi - 1}. \tag{28}$$

$$\sigma^*_{xz} = \tan 2\psi \frac{S\cot\phi - \sigma^*_{zz}}{\csc\phi\sec 2\psi - 1}. \tag{29}$$

Noncohesive Coulomb Wedge

- The following stress components satisfy the static momentum balance, the failure condition and the boundary conditions on the top surface:

$$\sigma^*_{xz} = (\rho - \rho_f) g z \sin\alpha, \tag{30}$$

$$\sigma^*_{zz} = -\rho_f g D - \rho g z \cos\alpha, \tag{31}$$

$$\sigma^*_{xx} = -\rho_f g D - \rho g z \cos\alpha \frac{\csc\phi \sec 2\psi_0 - 2\lambda + 1}{\csc\phi \sec 2\psi_0 - 1}, \tag{32}$$

provided that

$$\frac{\tan 2\psi_0}{\csc\phi \sec 2\psi_0 - 1} = \left(\frac{1 - \rho_f/\rho}{1 - \lambda}\right) \tan\alpha. \tag{33}$$

- (33) relates the stress orientation angle ψ_0 to the surface slope α; we have assumed that ψ_0 is constant and have made use of the relation $dD/dx = -\sin\alpha$.

Noncohesive Coulomb Wedge

- (30), (31), (32) are an exact solution for the state of stress in a sloping half-space on the verge of Coulomb failure.
- All that remains is to satisfy the basal boundary condition. Allowing for a different pore-fluid regime (like a different porosity) on the décollement fault, we have this boundary condition for the base:

$$\tau_b = -\mu_b(\sigma_n + p_{fb}), \tag{34}$$

  where p_{fb} is the pore-fluid pressure on the base and μ_b is the basal coefficient of friction.
- Expressing the basal shear and normal stress in terms of stress components in the $x - z$ coordinate system:

$$\tau_b = 1/2(\sigma^*_{zz} - \sigma^*_{xx}) \sin 2(\alpha + \beta) + \sigma^*_{xz} \cos 2(\alpha + \beta), \tag{35}$$

$$\sigma_n = \sigma^*_{zz} - \sigma^*_{xz} \sin 2(\alpha + \beta) - 1/2(\sigma^*_{zz} - \sigma^*_{xx})[1 - \cos 2(\alpha + \beta)]. \tag{36}$$

Noncohesive Coulomb Wedge

- (30), (31), (32), (35) and (36) are used to determine the dip of the surface on which the frictional sliding condition (34) is satisfied.
- After some algebra, β is given by

$$\alpha + \beta = \psi_b - \psi_0, \tag{37}$$

where

$$\frac{\tan 2\psi_b}{\csc\phi \sec 2\psi_b - 1} = \mu_b \left(\frac{1 - \lambda_b}{1 - \lambda} \right). \tag{38}$$

- (37) is the exact critical taper equation for a homogeneous noncohesive Coulomb wedge.

Noncohesive Coulomb Wedge

- Here is the usual procedure we take to play with the above critical taper equation.
  1. We can start from (38) to find ψ_b from μ_b and λ_b.

$$\frac{\tan 2\psi_b}{\csc\phi \sec 2\psi_b - 1} = \mu_b \left(\frac{1-\lambda_b}{1-\lambda} \right).$$

  2. Then, assume a value of the surface slope, α, to get ψ_0 from (33).

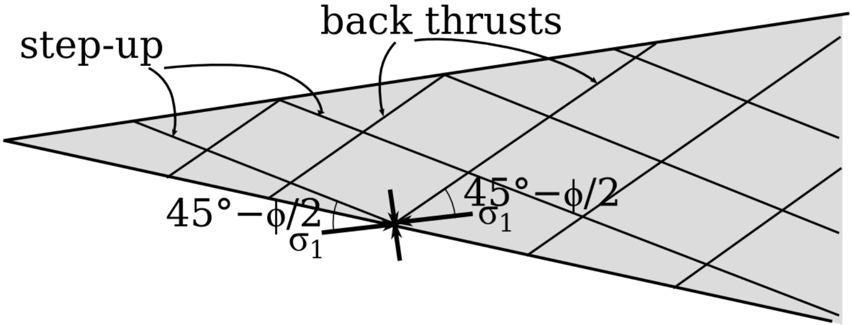
$$\frac{\tan 2\psi_0}{\csc\phi \sec 2\psi_0 - 1} = \left(\frac{1-\rho_f/\rho}{1-\lambda} \right) \tan\alpha.$$

  3. Since ψ_b, ψ_0 and α are known, we get β from the taper equation, (37).

$$\beta = \psi_b - \psi_0 - \alpha.$$

Noncohesive Coulomb Wedge

- Once we get all these angles, we can predict the orientation of step-up from the basal décollement fault as well as that of backthrust faults.



Noncohesive Coul

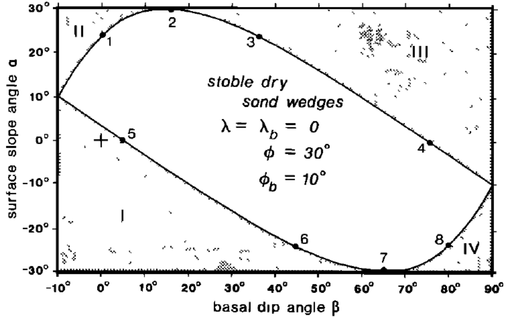


Fig. 9. Diagram showing stable and unstable regions of dry sand wedges having $\varphi = 30°$ and $\varphi_b = 10°$. Critical wedges labeled 1–8 are depicted in Figure 10.