Useful online resources

Dissemination of IT for the Promotion of Materials Science by Cambridge University

`https://www.doitpoms.ac.uk/index.php`

- Creep: Introduction
- Creep: Mechanisms
- Creep: Constitutive models
- Stress analysis and Mohr's circle
- Brittle fracture

Brittle and Ductile Deformations

- Rocks are elastic: they deform under loading and go back to the original shape (reference configuration) when the loading is removed.
- However, when the applied loading exceeds a certain critical value, rocks start to develop **permanent deformation.**
  - Linear and non-linear rheologies of rocks are used to describe permanent deformation as viscous flow.
  - Another mode of permanent deformation is fracturing: e.g., Faults and joints.
- We are going to study which mode becomes dominant under a given pressure and temperature condition. This understanding leads to a convenient way of representing both modes and required force in one plot: **yield strength envelope.**

Brittle Deformations

- Another related issue is what is the stress required for fracturing a rock and for causing the fractured surface to slip. Naturally, this question is related to the studies on faulting.
- Let's first get some intuition from these movies:
  1. Microscopic view to "permanent" deformation:
     `http://www.youtube.com/watch?v=n7LXYyohmgg`
  2. Tension tests on metals:
     `http://www.youtube.com/watch?v=D8U4G5kcpcM`

Brittle Deformations

- Stress-strain curves for linear and non-linear elastic materials.

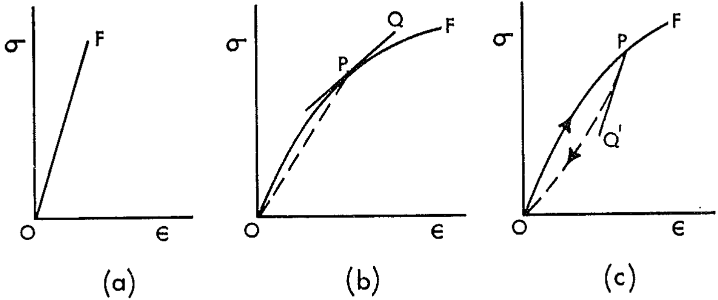


Fig. 4.2.1 (*a*) Linearly elastic material. (*b*) Perfectly elastic material showing tangent modulus *PQ* and secant modulus *OP*. (*c*) Elastic material with hysteresis, showing loading and unloading cycle.

(Ch. 4 in Fundamentals of Rock Mechanics, Jaeger, J. C. and N. G. W. Cook, 2nd ed., 1976, Chapman and Hall, London)

Brittle Deformations

- A typical stress-strain curve for rock.

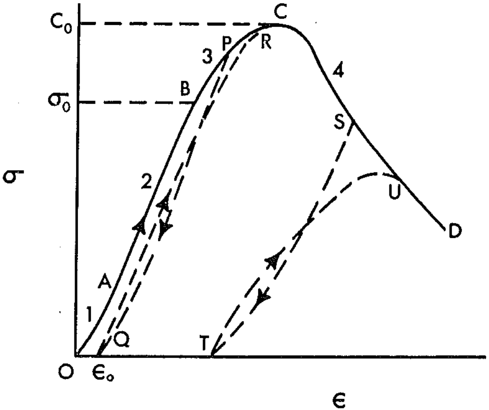


Fig. 4.2.2 The complete stress–strain curve for rock.

(Ch. 4 in Fundamentals of Rock Mechanics, Jaeger, J. C. and N. G. W. Cook, 2nd ed., 1976, Chapman and Hall, London)

Brittle Deformations

(cont'd)

- The curve has 4 regions
  1. OA: Slightly concave upward. Nearly elastic (i.e. no permanent deformation when unloaded) although maybe with hysteresis.
  2. AB: Almost linear. Nearly elastic although maybe with hysteresis.
  3. BC: Concave downward. Usually, $\sigma_0 \sim 2/3C_0$. Irreversible deformation starts in this region. Successive cycles of loading and unloading trace out different curves. For instance, paths PQ and QR leaves ϵ_0.
  4. CD: Failling region. Starts from the stress maximum, C. Characterized by a negative slope. Unloading (ST) leaves a large permanent deformation and reloading (TU) approaches the curve CD at a lower stress U than S.

Brittle Deformations

(cont'd)

- A material is said to be **ductile** under conditions in which it can sustain permanent deformation without losing its ability to resist load. The ductile state corresponds to the region BC.
- A material is said to be **brittle** under conditions in which its ability to resist load decreases with increasing deformation. The brittle state corresponds to the region CD.
- If loading is cyclic, a reloading will attain a higher stress (as in PQR) if the material is in a ductile state while a lower stress (as in STU) if in a brittle state.
- The process of **failure** is regarded as a continuous one which occurs progressively throughout the brittle region CD, in which the rock steadily deteriorates. The stress value at C, C_0 is called *uniaxial compressive strength*.

Brittle Deformations

(cont'd)

- In actual testing, sudden failure often occurs at some point of the curve CD with **complete loss of cohesion across a plane** and this is known as **brittle fracture.** For instance (and for fun), watch `http://www.youtube.com/watch?v=PRF_ZJZksp4`.
- The point B at which the transition from elastic to ductile behavior takes place is known as the **yield point** and the corresponding stress σ_0 as the **yield stress.**

Brittle and Ductile Deformations

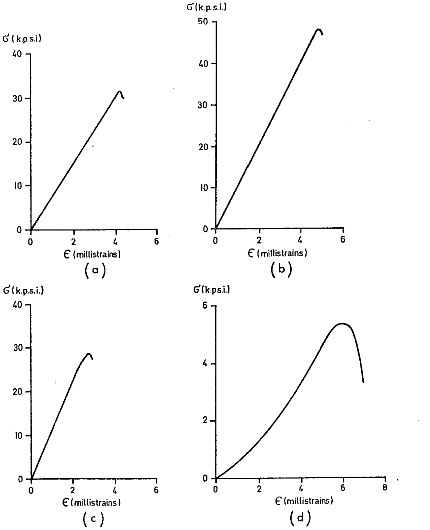


Fig. 4.2.3 Stress–strain curves for uniaxial compression in a stiff testing machine. (*a*) Solenhofen limestone. (*b*) Karroo dolerite. (*c*) Rand quartzite. (*d*) Gosford sandstone

- Some stress-strain curves of typical rocks for comparison with the idealized curve discussed earlier.
- These curves show how unimportant the regions OA and BC are in the practical cases.
- So, the assumption of linear elasticity up to failure is a good one in many cases.

Brittle and Ductile Deformations

< Effects of Confining Pressure >

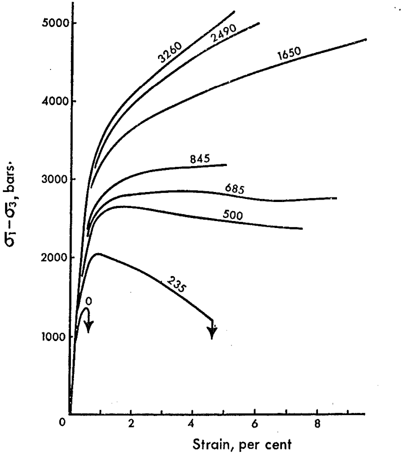


Fig. 4.3.2 Stress–strain curves for Carrara marble (after von Karman) at various confining pressures. The numbers on the curves are confining pressures in bars.

- Confining pressure up to 500 bars (50 MPa), brittle fracture occurs as before with an increase of strength and a small increase in permanent strain.
- The curve for 685 bars is completely different: It's a curve for a ductile state.
- The curve for 235 bars shows transitional behavior.
- Such a transition is called the **brittle-ductile transition**.

Brittle and Ductile Deformations

< Effects of Temperature >

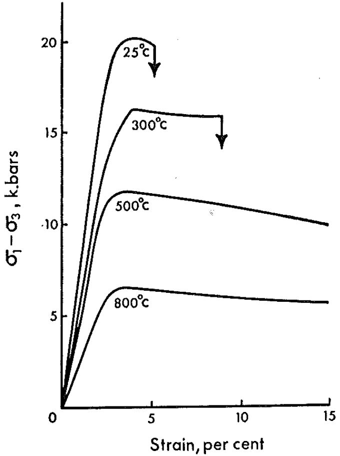


Fig. 4.3.3 Stress–strain curves for granite at a confining pressure of 5 kilobars and various temperatures (after Griggs, Turner, and Heard).

- The effect of increasing the temperature is to lower the brittle-ductile transition pressure.
- At room temperature, the brittle failure occurs.
- At higher temperatures, the substantial amount of permanent deformation is introduced without loss of load.

Brittle and Ductile Deformations

- Even the pervasively fractured rocks have a finite yield stress under confining pressure.
- Byerlee showed that the amount of shear stress (τ) required for the most favorably oriented fracture surface (or fault plane) is a linear function of the normal stress (σ_n) on that plane. This shear stress can be identified with the yield stress (τ_Y).
- Moreoever, the linear relationship between τ_Y and σ_n holds regardless of rock type.
- Since the normal stress can be related to lithostatic pressure[1], $\rho g z$, this fact leads to a linear relationship between the yield stress of pervasively fractured brittle rock and depth: i.e., $\tau_Y \propto z$

[1] The details will be worked out in the next class.

Ductile Deformations

- We can apply a general power-law rheology to ductile deformation. Then, for a given geotherm and an assumed value of strain rate, the shear stress becomes a function of depth, too.

$$\dot{\epsilon} = A\left(\sigma_1 - \sigma_3\right)^n \exp\left(-\frac{Q}{RT(z)}\right), \tag{1}$$

  where $(\sigma_1 - \sigma_3)$ can be related to the amount shear stress required for the assumed strain rate.

- If both brittle and ductile yield stresses are identified with $(\sigma_1 - \sigma_3)$, we can plot the stress difference $(\sigma_1 - \sigma_3)$ as a function of depth.
- At each depth, two stress difference values will be available: one for the brittle strength and the other for ductile strength. By picking the lower one, we get a single profile of strength, which is called the **yield strength envelope**.

Ductile Deformations

Table 6.4

$$\dot{\varepsilon} = A_p(\sigma_1 - \sigma_3)^n \, e^{-Q_P/R_g T}$$

Steady State Flow Properties for Selected Rocks and Minerals

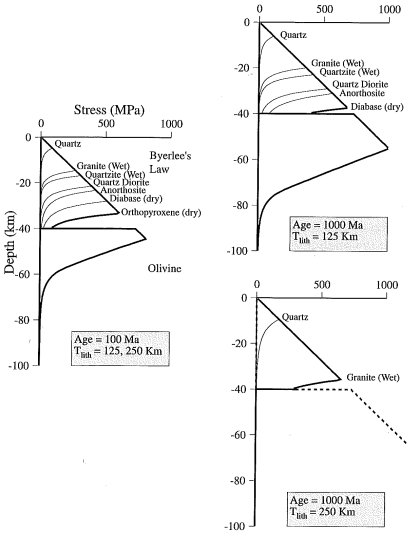
Rock/Mineral	Exponent n	A_p (Pa^{-n} s^{-1})	Q_p (kJ mol^{-1})	Primary Reference
Quartz	3.0	1.2×10^{-24}	92	Heard and Carter (1968)
Quartzite (dry)	3.0	6.1×10^{-24}	190	Brace and Kohlstedt (1980)
Quartzite (wet)	1.9	1.2×10^{-13}	173	Hansen (1982)
Granite (Westerly: wet)	1.9	7.9×10^{-16}	141	Hansen and Carter (1983)
Granite (Westerly: dry)	3.3	3.1×10^{-26}	186	Hansen and Carter (1983)
Anorthosite	3.2	3.2×10^{-22}	238	Shelton and Tullis (1981)
Diabase (dry)	3.05	3.1×10^{-20}	276	Caristan (1982)
Diabase (Columbia)	4.7	1.1×10^{-26}	488	Mackwell et al. (1998)
Diabase (Maryland)	4.7	5.0×10^{-28}	482	Mackwell et al. (1998)
Quartz Diorite	2.4	1.2×10^{-16}	212	Hansen (1982)
Orthopyroxene (wet)	2.8	1.0×10^{-19}	271	Rayleigh et al. (1971)
Orthopyroxene (dry)	2.4	1.2×10^{-15}	293	Ross and Nielsen (1978)
Clinopyroxene (wet)	3.3	2.3×10^{-14}	490	Boland and Tullis (1986)
Clinopyroxene (dry)	5.3	1.6×10^{-36}	380	Boland and Tullis (1986)
Olivine	3.0	7.0×10^{-14}	520	Goetze (1978)
Olivine (dry)	3.5	2.4×10^{-16}	540	Karato et al. (1986)
Olivine (wet)	3.0	1.9×10^{-15}	420	Karato et al. (1986)
Dunite(wet)	4.5	4.0×10^{-25}	498	Chopra and Paterson (1981)
Dunite(dry)	3.6	7.9×10^{-18}	535	Chopra and Paterson (1981)

(Isostasy and Flexure of the Lithosphere, Watts, A. B., 2001, Cambridge University Press, Cambridge)

Yield Strength Envelope

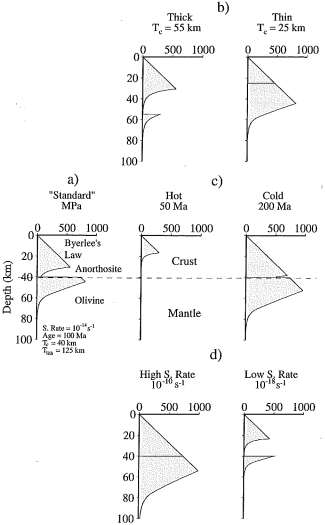
< YSE for continental lithosphere with $\dot{\varepsilon} = 10^{-14}\ \mathrm{s}^{-1}$ >

(Fig. 6.28 in Isostasy and Flexure of the Lithosphere, Watts, A. B., 2001, Cambridge University Press, Cambridge).

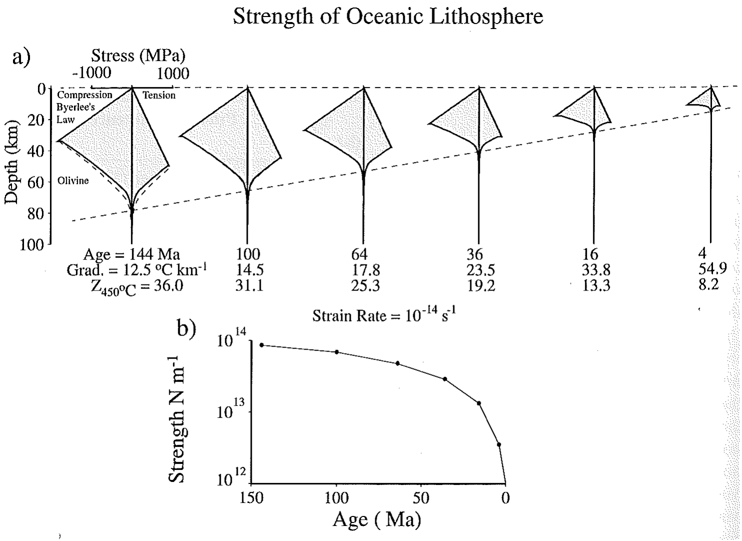


Yield Strength Envelope

(cont'd) (Fig. 6.29 in Isostasy and Flexure of the Lithosphere, Watts, A. B., 2001, Cambridge University Press, Cambridge)



Yield Strength Envelope



Mantle Rheology

- We studied incompressible, slow, steady-state Newtonian fluid quite thoroughly.
  - 1D channel flows.
  - 2D flows described by the stream function.
  - Thermal convection: linear stability analysis (episodic transient convection, steady-state convection.)
- We also learned non-linear rheologies for **subsolidus flow** of rocks: Diffusion creep and dislocation creep.
- So, what do all of these mean for the Earth? Convecting mantle has a Newtonian or a non-linear rheology?
- To your disappointment, we don't know. We can only make informed guess.

Mantle Rheology

- This general form of the $\dot{\epsilon} - \sigma$ relationship is valid for both diffusion and dislocation creep:

$$\begin{aligned}\dot{\epsilon} &= A\left(\frac{\sigma}{G}\right)^n \left(\frac{b}{h}\right)^m \exp\left(-\frac{E_a + pV_a}{RT}\right) \\ &= \eta_{\text{eff}}\,\sigma.\end{aligned} \tag{2}$$

  where A is the preexponential factor, G is the shear modulus, h is the grain size, b is the lattice spacing and η_{eff} is the effective viscosity.
- For diffusion creep, $n = 1$ and $m = 2.5$;
  for dislocation creep, $n = 3.5$ and $m = 0$.
- With lots of simplifications, one can make a deformation map ($\sigma - T$ plot for several values of $\dot{\epsilon}$) or $\mu_{eff} - T$ plot for several values of σ. See Fig. 7-20 and 7-21.
- On these plots, the typical **upper mantle** conditions fall on the **dislocation** creep region .

Mantle Rheology

- As far as the mantle is concerned, it is not crucial whether its rheology is Newtonian or a power law with $n \approx 3$.
- The reason is that the temperature and pressure dependence of viscosity, coming from $\exp\left(\frac{E_a+pV_a}{RT}\right)$, dominates any effects from stress dependence.
- Constraints from the surface observations are often not conclusive. Let's review the arguments given in Sec. 7-6 of T&S (2014, 3rd ed.)
  - In the non-Newtonian case, the strain rate and stress of postglacial rebound define $\mu_{eff,rebound}$ that is $1/3$ of the $\mu_{eff,convection}$ associated with mantle convection.
  - Considering uncertainties involved in the postglacial rebound studies, a factor of 3 is not too serious.

Mantle Rheology

- Another constraint comes from lab experiments of creep.
- Dry olivine at 1400°C shows stress-strain rate relations that fit well the cubic power-law rheology (Fig. 7-19):

$$\dot{\epsilon}_{xx} = -\dot{\epsilon}_{yy} = C_1 \sigma^3 e^{-E_a/RT}. \tag{3}$$

- Caveats: 1. C_1 was treated as a constant although temperature-dependent in principle. 2. Strain rate in lab expriments $\sim 10^{-8}$ s^{-1}, about 7 orders of magnitude larger than mantle strain rates.
- Nevertheless, the theoretical basis for the cubic power law seems reasonably sound.

Mantle Rheology

- In the previous chapter, we studied the convection in the mantle assuming the mantle is a Newtonian fluid.
- With lab experiments suggesting a cubic power law, what would be the implication of this non-linear rheology for convection?
- Again, it must be the temperature-dependence, not stress-dependence, that matters most.
- One of the effects of temperature-dependent viscosity we didn't consider is the **rigidity** of lithosphere (cold boundary layer) **inhibiting subduction.**
  - One evidence is in the much larger aspect ratios ($>> 1$) of convection cell inferred from major tectonic plates than the one predicted for a constant viscosity (~ 1).

Mantle Rheology

- Strong viscosity variation anticipated if there are thermal boundary layers elsewhere in the mantle, for instance, at the CMB. Upwelling of material in a narrow manatle plume is made possible by the lowered viscosity in it.
- If convection occurs in a layered fashion, there must be a thermal boundary layer between upper and lower mantle, implying the lower mantle is much hotter and therefore has a much lower viscosity.
  - Post-glacial rebound data, however, suggest an almost uniform mantle viscosity, which is understood in terms of strong and opposite dependence on temperature and pressure:

$$\mu \sim \exp\left(\frac{E_a + pV_a}{RT}\right). \tag{4}$$

  - Increase in T with depth reduces viscosity but the increase of pressure with depth increases viscosity.

Mantle Rheology

- The existence of asthenosphere, a zone decoupling lithosphere from the underlying mantle, is also suggested by the exponential dependence of visocisty on the inverse of temperature (see Fig. 7-22).
- Finally, the mantle cools at a relatively slow rate (see Sec. 7-8) because its temperature is buffered by the strong temperature dependence of is viscosity.
  - Decrease in heat production rate –> decrease in mantle temperature –> increase in viscosity –> decrease in Ra ($\propto 1/\mu$) –> decrease in convective heat flux (Nu $\propto$ Ra).