

Heat Energy I

Why do we care about Heat Energy?

- ▶ Heat transfer drives plate tectonics, climate change and many other changes.
- ▶ Material properties depend on temperature.
- ▶ What is heat?
A form of energy that is transferred from one body to another through thermal interactions. Note that “transfer” is the essential part of the definition. So, terms like heat transfer, heat conduction and heat flux are more often used than heat itself.
- ▶ Often *thermal energy* is distinguished from heat. Thermal energy is defined, without involving “transfer,” as the portion of the internal energy of a system that is responsible for the temperature of the system.

Heat Energy II

- ▶ In statistical physics, thermal energy is the sum of mechanical kinetic energy of particles.
<https://youtu.be/9joLYfayee8>
- ▶ Changes driven by heat transfer are felt by change in “warmth” and the degree of warmth is quantified by temperature.
- ▶ However, temperature is not the only variable that describes a “thermodynamic” system.
- ▶ In the classical thermodynamics, at least 4 *state variables* are considered: temperature, entropy, pressure and volume.
- ▶ Like potential energy in mechanics, there are *thermodynamic potentials*, which represent a capacity for driving changes under various situations.

Heat Energy III

- ▶ The four fundamental ones are internal energy, enthalpy, Gibbs free energy and Helmholtz free energy.

- ▶ Internal energy (U) : Independent variables: s , V .

Helmholtz free energy (A) : $A = U - TS$. The portion of internal energy that can do mechanical work.
Independent variables: T , V .

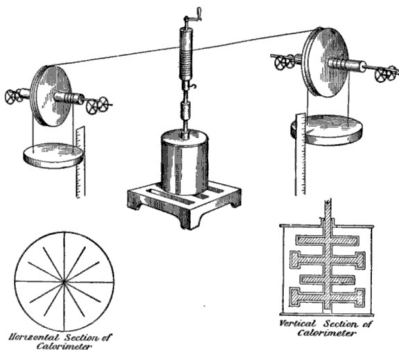
Enthalpy (H) : $H = U - pV$. The portion of internal energy that can be released as heat.
Independent variables: s , P .

Gibbs free energy (G) : $G = U - TS - pV = H - TS$. The portion of internal energy that can drive internal change like phase transition.
Independent variables: T , p .

Heat Energy IV

- ▶ As stated in its definition, thermal energy is a part of the internal energy but of the other three energies as well because the four potentials and the four state variables are all inter-related.
- ▶ Since we don't have time to derive the relations rigorously, let's just feel the complex behaviors of thermodynamic systems through some examples.
- ▶ Joule's experiments for measuring "thermochemical calorie" ($\approx 4.2 \text{ J/cal}$): mechanical work \rightarrow friction between water and paddle \rightarrow increase in water's thermal energy

Heat Energy V



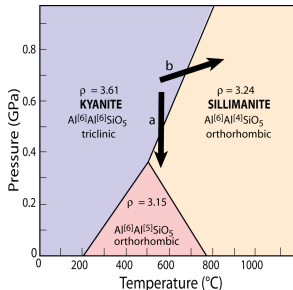
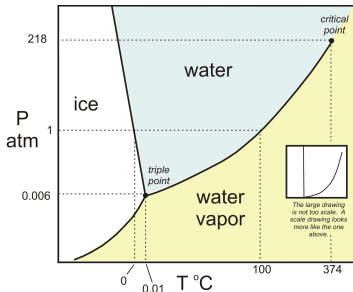
Joule's Water-Churning Apparatus for Determining the Mechanical Equivalent of Heat.



- ▶ Bending a coat hanger: mechanical energy \rightarrow internal friction in metal \rightarrow increase in metal's thermal energy.
- ▶ Stretching a rubber band: mechanical energy \rightarrow entropy decrease \rightarrow thermal energy to lose.

Heat Energy VII

- ▶ phase transition: water, aluminosilicates, metamorphic facies.



http://serc.carleton.edu/research_education/equilibria/phaserule.html

http://serc.carleton.edu/research_education/equilibria/PTPaths.html

heat transfer → change in bond between particles (e.g., liquid to gas) or in the arrangement of particles (solid-solid transformation).

Heat Energy VIII

- ▶ For our purpose, the terms, heat and thermal energy, are interchangeable so we will use heat or heat energy as meaning thermal energy from now on.
- ▶ Now, let's learn how to quantitatively understand heat transfer.
- ▶ For simplicity, (1) we consider a body and only heat energy in it.
- ▶ Deformation of the body means some work must have been done to the body and/or by the body, which will lead to change in internal energy. So, (2) we do not consider deformation here.
- ▶ We further assume that (3) there is no heat energy sink or source for now. These can be easily added when necessary.
- ▶ Finally, (4) we consider only heat transfer by *conduction*.

Heat Energy IX

- ▶ *Heat capacity at constant pressure, C_p :*

$$C_p = \left(\frac{\partial Q}{\partial T} \right)_p, \quad (1)$$

where Q is the heat energy.

- ▶ *Specific heat capacity at constant pressure, c_p :*

$$c_p = \frac{C_p}{m} \quad (2)$$

where m is mass.

- ▶ *Heat energy per mass, q :*

$$q = \int_0^T c_p(T) dT \quad (3)$$

or if c_p is not a function of temperature,

$$q = c_p T. \quad (4)$$

Heat Energy X

- ▶ *Heat energy of a body, Q :*

$$Q = \int_V \rho c_p T dV. \quad (5)$$

- ▶ Under the set of assumptions listed above, the law of energy conservation states that the time rate of change of heat energy within a body is equal to the net flux of heat energy through its boundaries:

$$\frac{d}{dt} \int_V \rho c_p T dV = - \int_{\partial V} \mathbf{f} \cdot \mathbf{n} dS, \quad (6)$$

where \mathbf{f} is heat flux, representing heat energy flowing through unit area per unit time.

Heat Energy XI

- ▶ *Fourier's law of heat conduction:*

$$\mathbf{f} = -k\nabla T, \quad (7)$$

where k is *heat conductivity*.

- ▶ The energy conservation equation becomes

$$\frac{d}{dt} \int_V \rho c_p T dV = \int_{\partial V} k \nabla T \cdot \mathbf{n} dS. \quad (8)$$

- ▶ By applying the divergence theorem to the r.h.s and bringing the time derivative into the integral on the l.h.s, we get

$$\int_V \frac{\partial}{\partial t} (\rho c_p T) dV = \int_V \nabla \cdot (k \nabla T) dV. \quad (9)$$

Note that material time derivative is identical to partial time derivative since spatial velocity is zero.

Heat Energy XII

- ▶ Let's further assume that material properties, ρ , c and k are constant.

$$\int_V \rho c_p \frac{\partial T}{\partial t} dV = \int_V k \nabla^2 T dV. \quad (10)$$

- ▶ Since the energy conservation should be true for any arbitrary neighborhood around a point in the body,

$$\int_V \left(\rho c_p \frac{\partial T}{\partial t} - k \nabla^2 T \right) dV = 0 \quad (11)$$

for an arbitrary V , meaning the integrand should be identically zero.

- ▶ We finally arrive at the familiar form of the “heat equation”:

$$\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T. \quad (12)$$

Heat Energy XIII

- ▶ Note that the left hand side of (8) could have been more complicated according to Reynold's transport theorem. It wasn't because of our assumption that the body doesn't deform.
- ▶ Let's work out the units of the thermal properties.
- ▶ heat flux (= energy per unit area per unit time)?
- ▶ heat conductivity k ?
- ▶ specific heat capacity at constant pressure c_p ?
- ▶ what about $k/\rho c_p$ in

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \nabla^2 T ? \quad (13)$$

This is called *heat diffusivity*.

Measuring the Earth's heat flow

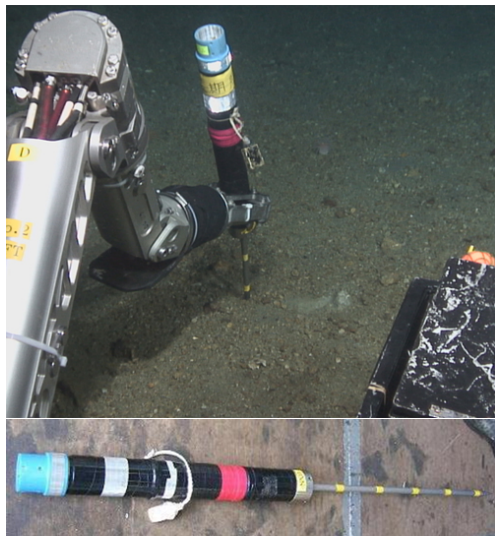
- ▶ Fourier's law tells us that we need to know two things to calculate heat flux: temperature gradient and heat conductivity.
- ▶ Measuring heat conductivity of rock (see Fig. 4-3 of T&S).

Measuring temperature gradient

- ▶ On continents, it's been known since 1800s that temperature in caves and mines increases with depth
 $dT/dy = 20 - 30 \text{ K/km}$
- ▶ Conductivity of the near-surface rocks are 2-3 W/m/K.
- ▶ So, heat flow is 40-90 mW/m².
- ▶ However, measurements in deep drill holes are necessary in order to exclude near-surface transient variation in temperature. At least 300 m-deep holes are required. We'll learn later what those time-dependent temperature variations are and how deep they can penetrate.
- ▶ Still, it's not easy to measure temperature gradient that correspond solely to the Earth's steady state temperature field (influenced by drilling fluid or groundwater).

Measuring temperature gradient on the ocean floor

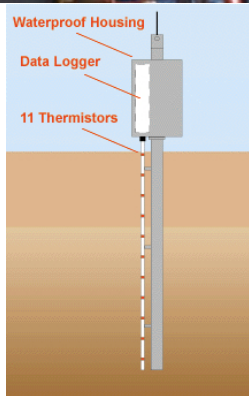
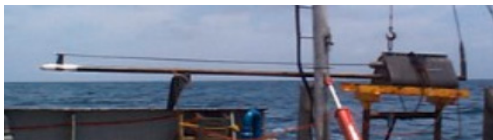
60 cm-long probe



http://www.jamstec.go.jp/e/about/press_release/20121011/

Measuring temperature gradient on the ocean floor

~3 m-long probe



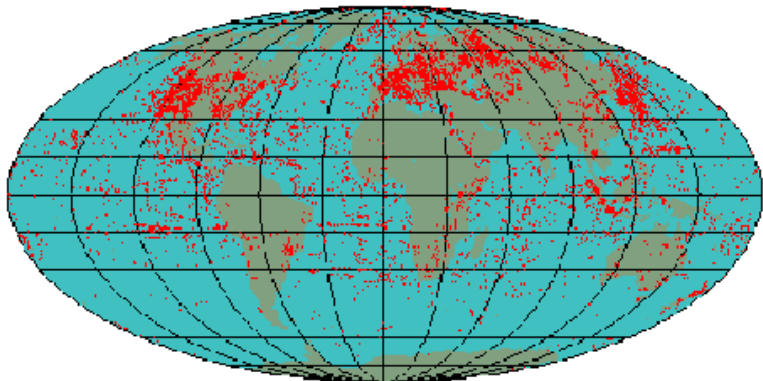
http://www.tdi-bi.com/field_services/hf_info/description.htm

Measuring temperature gradient on the ocean floor

- ▶ The near-surface heat flow in the oceanic crust is almost a constant because the sea water in the deep oceans is free from climatic influences.
- ▶ The constant temperature of sea water is between 1 and 2 °C at which sea water becomes the densest. The variation is due to that of salinity.
- ▶ However, hydrothermal circulation can still influence the measurements.

Measuring the Earth's heat flow

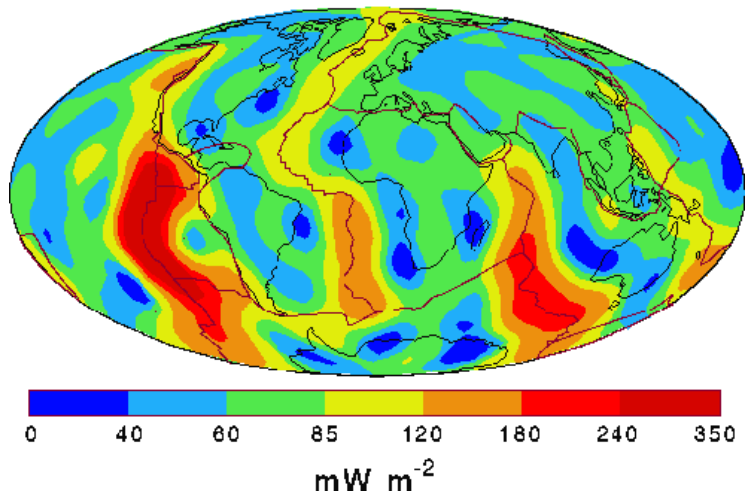
Heat Flow Sites



(Pollack et al., Rev. Geophys., 1993)

Measuring the Earth's heat flow

Heat Flow



(Pollack et al., Rev. Geophys., 1993)

Measuring the Earth's heat flow

- ▶ The mean heat flow for all continents is $65 \pm 1.6 \text{ mW m}^{-2}$.
 - ▶ High heat flow regions on the continents are correlated with active volcanic region.
 - ▶ Continental heat flow \propto surface concentrations of the radioactive isotopes. More on this later.
- ▶ The mean measured heat flow for all the oceans is $101 \pm 2.2 \text{ mW m}^{-2}$.
 - ▶ Negligible contribution from radiogenic isotopes: Oceanic crust has an order of magnitude lower concentration of radiogenic isotopes and is 5 times thinner than continental crust.
 - ▶ Oceanic heat flow \propto the age of the sea floor. More on this later.

Measuring the Earth's heat flow

- ▶ The rate of total heat energy loss from the interior of the Earth = $f_c A_c + f_o A_o$

$$\text{Continents: } 65 \text{ mW m}^{-2} \times 2 \times 10^8 \text{ km}^2 = 1.3 \times 10^{13} \text{ W}$$

$$\text{Oceans: } 101 \text{ mW m}^{-2} \times 3.1 \times 10^8 \text{ km}^2 = 3.13 \times 10^{13} \text{ W.}$$

- ▶ The global mean heat flux is

$$4.43 \times 10^{13} \text{ W} / (5.1 \times 10^8 \text{ km}^2) = 87 \text{ mW m}^{-2}$$